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Algebraic Structures and Practical Implications of Interval-Valued Fermatean Neutrosophic Super HyperSoft Sets in Healthcare

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ABSTRACT

Healthcare workers, including doctors and medical staff, must consistently make informed decisions that significantly affect patients on individual, community, national, and global scales. Healthcare practitioners must occasionally make judgments with constrained information, resources, and knowledge, yet it is anticipated that these choices are meticulously measured and precise. Familiarizing oneself with precise concepts pertaining to medical decision-making is essential. The complexity of decision-making (DM) arises from the environment's ambiguous, imprecise, and uncertain characteristics, especially when several attributes are present and further categorized. We have used the hypersoft set concept to address such complex challenges. This article defines interval-valued Fermatean neutrosophic super hyper-soft sets. Its fundamental operations are subsets, equality, null sets, complements, unions, and intersections. Using these algebraic procedures, we establish a numerical example for the prioritization of patients awaiting organ transplantation. This hybrid environment may effectively manage uncertainty and yield distinctive results.

1. Introduction

We acknowledge the inherent ambiguity of medical practice. Experienced physicians often experience uncertainty, especially regarding complex co-morbid medical conditions that may hinder the

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application of contemporary medical knowledge. Addressing medical uncertainty presents a fundamental challenge that trainees must manage throughout their developmental process [1]. Insufficient capacity to manage medical ambiguities can disrupt medical training and potentially jeopardize patient welfare [2]. Physicians exhibiting intolerance to uncertainty often order an excessive number of diagnostic tests and demonstrate reluctance in communicating their uncertainties to patients during the decision-making process [3]. Inadequate management of uncertainty can lead to unnecessary anxiety and distress in patients. Medical students often experience medical ambiguity during their clerkship, viewing it as a major source of stress. Furthermore, medical students who experience difficulty with ambiguity often exhibit negative attitudes towards marginalized populations [4]. Residents encounter challenges in clinical reasoning due to complex environmental factors. Recognizing and addressing medical uncertainty, as well as developing effective coping techniques, are considered fundamental clinical competencies for medical graduates and trainees in Scotland, Australia, the European Union, the United Kingdom, and the United States [5]. Future medical practices may leverage advanced technology and precision medicine, including sophisticated information technology and artificial intelligence, to reduce uncertainty in the medical field. Emerging knowledge and future technologies may introduce unforeseen uncertainties in medical practice [6]. Despite advancements in the fourth industrial revolution, uncertainty in medicine may persist or increase. Therefore, medical academia and training institutions should prepare medical students and trainees to effectively navigate ambiguity [7].

In 1965, Zadeh established the theory of fuzzy sets and the concept of membership to mitigate uncertainty [8]. This principle, broadly acknowledged in modern discussions, supports the convenience and comfort encountered in everyday living. This theory enabled the progression of diverse set structures, such as single-valued fuzzy sets (SVFS), multiple-valued fuzzy sets (MVFS) [9], bipolar fuzzy sets (BFS) [10], interval-valued fuzzy sets (IVFS) [11], m-polar interval-valued fuzzy sets (m-PIVFS) [12], and fuzzy rough sets (FRS) [13]. In 1986, Atanassov advanced the notion of fuzzy sets (FS) by proposing intuitionistic fuzzy sets (IFS), which encompass both membership and non-membership values [14]. Zhang and Xu [15] broadened the notion of IFS to encompass Pythagorean fuzzy sets (PFS) and formulated operational principles and decision-making techniques for addressing MCDM issues. In 1998, Smarandache introduced a new concept to tackle uncertain, inconsistent, and indeterminate situations, known as neutrosophic sets (NS) [16]. NS incorporates indeterminacy values in conjunction with membership and non-membership values (T, I, and F), which function independently of each other. Neutrosophy (NS) has been expanded to include constructs such as bipolar neutrosophic sets (BPNS) [17], single-valued neutrosophic sets (SVNS) [18], multi-valued neutrosophic sets (MVNS) [19], interval-valued neutrosophic sets (IVNS) [20], and multi-valued interval neutrosophic sets (MVINS) [21], as designated by decision-makers (DM) utilizing neutrosophic numbers. The concepts have significant implications in practical applications, particularly in multi-criteria decision-making (MCDM) challenges [22-24]. Various techniques have been proposed by scholars to address multi-criteria decision-making (MCDM) in different environments [25-26]. Mathematical methods serve as powerful tools for tackling real-world challenges, such as human resource selection, device selection, shortest path identification, robot selection, security assessments, medical equipment choices, and environmental safety strategies [27-30]. In response to the limitations and challenges associated with existing set architectures, Molodstov [31] introduced the concept of a soft set (SS). Maji [32] enhanced the concept of soft sets by merging it with neutrosophic sets, leading to the creation of neutrosophic soft sets (NSS) to address challenges related to indeterminacy. Deli [33] presented interval-valued neutrosophic soft sets (IVNSS), outlining essential concepts, operations, and methodologies for decision-making. Alkhazaleh presented the concept of n-valued refined neutrosophic soft sets (nVNRSS) [34]. A hypersoft set (HSS) was first introduced by Smarandache in 2018 [35]. The set is characterized as a mapping from the specified set of attributes and the power set of the universal set to the Cartesian product of attributes, which are subsequently divided further. Various extensions have been proposed, such as fuzzy hypersoft sets (FHSs), intuitionistic hypersoft sets (IHSs), and neutrosophic hypersoft sets (NHSs), to address different levels of truth, uncertainty, and indeterminacy [36]. IHSs, and NHSs operations and its applications are developed by many researchers that highlight the worth of the work [37-40]. A comprehensive examination of aggregation operators, similarity, and distance measures, as well as their application in DM settings for IVNHSs, was discussed in [41-44]. The contributions played a crucial role in enhancing the understanding and application of IVNHSs across various domains. The concepts of CC and WCC for IVNHSs were presented by Zulqarnain et al. [45]. Researchers devolped many other usefull operations for the refined neutrosophic logic to handle the uncertainty [46-49]. To address the shortcomings of the current research, we present the Interval-Valued Fermatean Neutrosophic Super HyperSoft sets, a hybrid structure capable of managing uncertainty with significant efficacy. The suggested method demonstrates adaptability by offering a mathematical framework to evaluate and aggregate several alternatives in multiple real-world contexts, including environmental sustainability, student internship selection, and healthcare facility selection. Each criterion has been meticulously assessed and categorised into several levels, from low to high, to provide a comprehensive study of all ideas. Through diverse real-world examples, including definitions of operators and their tabular representations, we have elucidated the practical implications of IVFN-SHS sets for selecting highly profitable projects among many options.

2. Preliminaries

Definition 1: Interval-Valued Fermatean Neutrosophic Soft Set (IVFN-SS)
Suppose we have a non-empty universal set “$\mathfrak{X}$”, “P($\mathfrak{X}$)” is the family of Interval-Valued Fermatean Neutrosophic values and a set of attributes is “$\mathfrak{A}$” related to “$\mathfrak{X}$”. Also let “$\mathfrak{H}$” be the subset of “$\mathcal{R}$” and the relation $\mathfrak{F}$:$\mathfrak{H} \to$ P($\mathfrak{X}$). The pair ($\mathfrak{F}, \mathfrak{H}$) is referred as the interval-valued fermatean neutrosophic soft set over “$\mathfrak{X}$”.

Definition 2: Hyper Soft Set (HSS)
Suppose we have a non-empty universal set “$\mathfrak{X}$”, “P($\mathfrak{X}$)” is its power set and a set of attributes is “$\mathfrak{A}$ = $\{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n\}$” for n $\geq$ 1 related to “$\mathfrak{X}$”. In addition, the corresponding attribute sets are $\{\beta_1, \beta_2, \beta_3, ..., \beta_n\}$ with $\beta_i \ \cap \ \beta_j$ = ϕ, for i,j $\in$ {1,2,3,...,n} and i $\neq$ j. Also, the function F: $\beta_1 \times \beta_2 \times \beta_3 \times ...\beta \times \ beta_n \to$ P($\mathfrak{X}$) and the combination ($F, \beta_1 \times \beta_2 \times \beta_3 \times ... \times \beta_n$) over "$\mathfrak{X}$" is called the HyperSoft set.

Definition 3: Interval-Valued Fermatean Neutrosophic HyperSoft Set (IVFN-HSS)

Suppose we have a non-empty universal set “$\mathfrak{X}$”, “P($\mathfrak{X}$)” is its power set and a set of attributes is “$\mathfrak{A}$ = $\{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n\}$” for n $\geq$ 1 related to “$\mathfrak{X}$”. In addition, the corresponding attribute sets are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \ \cap \ \beta_j$ = ϕ, for i,j $\in$ {1,2,3,...,n} and i $\neq$ j.
Also, the function J: $\beta_1 \times \beta_2 \times \beta_3 \times ... \times \beta_n \to$ P($\mathfrak{X}$) and
J($\beta_1 \times \beta_2 \times \beta_3 \times ... \times \beta_n$) = $\{< x, [T_M(J(x)), T_N(J(x))], [I_M(J(x)), I_N(J(x))], [F_M(J(x)), F_N(J(x))] > x \in \mathfrak{X}\}$ over "$\mathfrak{X}$" is called the Interval-Valued Fermatean Neutrosophic Hyper Soft set. Where "T" denotes the truth membership grade, "I" denotes the indeterminacy membership grade and "F" denotes the falsity membership grade, also M and N in the sub-script shows the membership degree and non-membership degree, respectively.

Definition 4: Interval-Valued Fermatean Neutrosophic Super HyperSoft Set (IVFN-SHSS)

Suppose we have a non-empty universal set “$\mathfrak{X}$”, “P($\mathfrak{X}$)” is its power set and a set of attributes is “A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$” for n $\geq$ 1 related to “$\mathfrak{X}$”. In addition, the corresponding attribute sets are

$\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j = \phi$, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the function J: $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) \rightarrow$ P($\mathfrak{X}$) and J($\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n)) = \{< x, [T_J(\varrho)^M(x), T_J(\varrho)^N(x)], [I_J(\varrho)^M(x), I_J(\varrho)^N(x)], [F_J(\varrho)^M(x), F_J(\varrho)^N(x)] > x \in \mathfrak{X}, \varrho \in (\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n)) \subseteq \beta_1, \beta_2, \beta_3, ..., \beta_n\}$ over "$\mathfrak{X}$". This is known as the Interval-Valued Fermatean Neutrosophic Super HyperSoft set. Where "T" denotes the truth membership grade, "I" denotes the indeterminacy membership grade and "F" denotes the falsity membership grade, also M and N in the sub-script shows the membership degree and non-membership degree, respectively.

Example 1:

For instance, let us take a critical and complex real-life scenario for the selection of a healthcare facility's location considering various attributes of the location and multiple criteria based on the people's needs and problems. Many factors had to be taken into account by the patients for the selection of healthcare facility, some of the main factors are treatment cost, technology and equipment available, best doctors, etc. For example, some of the facilities may be affordable but do not have advanced treatments and specialized doctors, while some facilities may provide specialist doctors and advanced technology but are highly expensive. In the same way, a healthcare facility close to the patient can save time for travel but may not have specialized doctors and advanced technology. Patients have to choose between these options such as convenience, treatment quality, traveling time, technology, cost, etc. Due to this healthcare facilities must be located at the best site and have every convenience patient can have. Therefore, the selection of the healthcare facility's location involves the evaluation of various attributes such as treatment cost, technology, specialist doctors, travel time, etc. The aim of this example is to select the best location considering the following main criteria,

- Travel time.
- Treatment cost.
- Advanced technology.
- Specialist doctors.
- Quality of care.

All of the above criteria are denoted by interval-valued fermatean neutrosophic HyperSoft numbers. Suppose two locations for the selection denoted by L_1 and L_2 are given as in Table 1,

For interval-valued fermatean neutrosophic Super HyperSoft set, we are going to access the locations on the following main and sub-criteria (as shown in the Table 2),

Travel time Traffic conditions and Emergency accessibility.

Treatment cost Consultation cost and Charges for medication and advanced surgeries.

Advanced technology Equipment availability and Surgical tools.

Specialist doctors Doctor's experience and Availability of expert.

Quality of care Cleanliness and Patient's satisfaction.

Table 1
Selection of Healthcare Facility Location using IVFN-HS

Criteria	L_1	L_2
Travel time.	$\begin{pmatrix}(0.50, 0.38),\\(0.02, 0.41),\\(0.39, 0.32)\end{pmatrix}$	$\begin{pmatrix}(0.30, 0.11),\\(0., 0.77),\\(0.08, 0.73)\end{pmatrix}$
Treatment cost.	$\begin{pmatrix}(0.56, 0.24),\\(0.21, 0.31),\\(0.65, 0.52)\end{pmatrix}$	$\begin{pmatrix}(0.55, 0.40),\\(0.22, 0.42),\\(0.32, 0.88)\end{pmatrix}$
Advanced technology.	$\begin{pmatrix}(0.97, 0.45),\\(0.54, 0.37),\\(0.75, 0.25)\end{pmatrix}$	$\begin{pmatrix}(0.86, 0.61),\\(0.37, 0.87),\\(0.32, 0.42)\end{pmatrix}$
Specialist doctors.	$\begin{pmatrix}(0.45, 0.45),\\(0.56, 0.72),\\(0.64, 0.19)\end{pmatrix}$	$\begin{pmatrix}(0.65, 0.85),\\(0.11, 0.74),\\(0.96, 0.93)\end{pmatrix}$
Quality of care.	$\begin{pmatrix}(0.98, 0.34),\\(0.17, 0.26),\\(0.58, 0.72)\end{pmatrix}$	$\begin{pmatrix}(0.33, 0.66),\\(0.77, 0.05),\\(0.28, 0.18)\end{pmatrix}$

3. Selection of the Most Prioritized Patient for Organ Transplant

It is always critical and complicated to choose between various patients suffering from end-stage organ failure, for organ transplantation in this modern healthcare era. For healthcare management, it is a challenge to allocate a healthy organ when the supply of the organ is limited but the number of patients has been increasing at an alarming rate.
In the process of transplantation, lack of the healthy donor organs is the most critical challenge for healthcare facilities. Additionally, imbalance between the supply and demand is based drastically on various factors such as aging populations, chronic diseases and a narrow number of healthy organs. It is also necessary for the donor organs to meet some strict conditions such as availability time, compatibility with the patient and health condition of the donor person, etc. further reducing the number of viable transplants. For instance, organs like the liver and heart have a small time outside the donor person's body obligating rapid surgical intercession and transplantation. On the other hand, diseases like liver cirrhosis, heart and kidney failures, etc. are growing rapidly with the time requiring transplantation of healthy organs as the only viable diagnosis expanding the need for donor organs on a higher level. It is very difficult for healthcare facilities to provide not only immediate transplantation to save the lives of the patients but also provide long-term results. Considering all these complex constraints and different criteria healthcare management faces a lot of difficulties in the selection of suitable recipients. This process will evaluate a list of different patients that satisfy the established standards with a detailed explanation of each individual for the selection. For the selection of the best candidate we have the following criteria,

- Organ compatibility
- Urgency level
- Age
- Health condition of patient

Table 2
Selection of healthcare facility location using IVFN-SHS.

Criteria	Sub-Criteria	L_1	L_2
Travel time.	Traffic condition	$\begin{pmatrix}(0.38,0.50),\\(0.48,0.98),\\(0.78,0.68)\end{pmatrix}$	$\begin{pmatrix}(0.32,0.39),\\(0.19,0.15),\\(0.25,0.35)\end{pmatrix}$
	Emergency accessibility	$\begin{pmatrix}(0.28,0.18),\\(0.16,0.37),\\(0.89,0.99)\end{pmatrix}$	$\begin{pmatrix}(0.58,0.28),\\(0.55,0.53),\\(0.57,0.10)\end{pmatrix}$
Treatment cost.	Consultation cost	$\begin{pmatrix}(0.47,0.27),\\(0.01,0.20),\\(0.49,0.64)\end{pmatrix}$	$\begin{pmatrix}(0.88,0.92),\\(0.72,0.42),\\(0.54,0.63)\end{pmatrix}$
	Charges for medication and advanced surgeries.	$\begin{pmatrix}(0.67,0.39),\\(0.37,0.45),\\(0.57,0.63)\end{pmatrix}$	$\begin{pmatrix}(0.44,0.43),\\(0.12,0.22),\\(0.37,0.23)\end{pmatrix}$
Advanced technology.	Equipment availability	$\begin{pmatrix}(0.32,0.39),\\(0.02,0.41),\\(0.88,0.33)\end{pmatrix}$	$\begin{pmatrix}(0.73,0.53),\\(0.75,0.74),\\(0.13,0.70)\end{pmatrix}$
	Surgical tools	$\begin{pmatrix}(0.71,0.83),\\(0.53,0.77),\\(0.23,0.61)\end{pmatrix}$	$\begin{pmatrix}(0.67,0.71),\\(0.41,0.02),\\(0.86,0.97)\end{pmatrix}$
Specialist doctors.	Doctor's experience	$\begin{pmatrix}(0.14,0.77),\\(0.25,0.66),\\(0.43,0.57)\end{pmatrix}$	$\begin{pmatrix}(0.16,0.16),\\(0.26,0.36),\\(0.11,0.31)\end{pmatrix}$
	Availability of expert	$\begin{pmatrix}(0.89,0.56),\\(0.72,0.65),\\(0.64,0.29)\end{pmatrix}$	$\begin{pmatrix}(0.39,0.49),\\(0.59,0.67),\\(0.78,0.21)\end{pmatrix}$
Quality of care.	Cleanliness	$\begin{pmatrix}(0.13,0.20),\\(0.42,0.10),\\(0.76,0.69)\end{pmatrix}$	$\begin{pmatrix}(0.29,0.35),\\(0.87,0.26),\\(0.16,0.25)\end{pmatrix}$
	Patient's satisfaction	$\begin{pmatrix}(0.94,0.72),\\(0.44,0.47),\\(0.40,0.60)\end{pmatrix}$	$\begin{pmatrix}(0.72,0.53),\\(0.81,0.53),\\(0.76,0.79)\end{pmatrix}$

- Expected quality of patient's life after transplantation

3.1 Established Standards for the Selection of the Most Prioritized Patient for Organ Transplant

We are going to review some of the main criteria that have a greater impact on the project than others, along with their availability in different suitable patients.

1. Organ compatibility:
    - Low: The donor organ is less compatible with the patient who meets less criteria, such as different tissue type or blood type, increasing these of rejection.
    - Medium: The donor organ satisfies most of the conditions for transplantation but faces

some challenges such as less tissue-type compatibility.
- High: The donor organ is fully aligned with the constraints of transplantation criteria.

2. Urgency level:
    - Low: The patient can wait for a perfect donor organ as their condition is stable.
    - Medium: The patient's condition is critical but not life threatening hence moderate emergency actions should be taken.
    - High: Urgent transplantation is necessary because patient's condition is life threatening.
3. Age:
    - Low: The patient is old aged hence the expected outcomes will be limited.
    - Medium: The patient has moderate age and offer balanced outlook for potential results.
    - High: Ideal candidate for transplantation as they are younger and have less complicated diseases and strong immune systems offering a longer life expectancy.
4. Health condition of patient
    - Low: The patient is suffering from other fatal diseases making them a less suitable candidate for transplantation.
    - Medium: The patient suffers from some diseases but they can recover from them in time.
    - High: The patient does not suffer from any other disease which can affect his immune system making them perfect for the selection.
5. Expected quality of patient's life after transplantation
    - Low: The patient has less chances of survival after transplantation because of their history of defiance of medical treatments.
    - Medium: The patient has moderate chances of survival with some manageable risks which can be neglected.
    - High: The patient has minimal health risks and has excellent prognosis.

4. Multi-Criteria Decision Making Using IVFN-SHS Set Under Uncertain Conditions

The evaluation team assigned by the healthcare facility has to consider all the patients against the multiple criteria while keeping in view different types of uncertainties and vagueness in the data such as life expectancy, age, health condition, urgency level, etc. For instance, evaluation team may prioritize a patient having high organ compatibility, high urgency level instead of the project having medium age and low expected quality of patient's life after transplantation. This procedure allows the evaluation team to choose the most suited patient which aligns with the criteria of the healthcare facility ensuring the donor organ is allocated to the most suitable patient.
Suppose "$\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$" be the family of proposed projects, also the set of criteria is "$\vartheta = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5\}$" and their respective attributes are elaborated below,

1. ϑ_1 = Organ compatibility = $\{$Low Organ compatibility (A_1), Medium Organ compatibility (A_2), High Organ compatibility (A_3)$\}$

2. ϑ_2 = Urgency level = $\{$Low Urgency level (B_1), Medium Urgency level (B_2), High Urgency level (B_3)$\}$

3. ϑ_3 = Age = $\{$Low Age (C_1), Medium Age (C_2), High Age (C_3)$\}$

4. ϑ_4 = Health condition of patient = $\{$Low Health condition of patient (D_1), Medium Health condition of patient (D_2), High Health condition of patient (D_3)$\}$

5. ϑ_5 = Expected quality of patient's life after transplantation = $\{$Low Expected quality of patient's life after transplantation (E_1), Medium Expected quality of patient's life after transplantation (E_2), High Expected quality of patient's life after transplantation (E_3)$\}$

Suppose $\mathfrak{W} : \vartheta_1 \times \vartheta_2 \times \vartheta_3 \times \vartheta_4 \times \vartheta_5 \rightarrow \vartheta_n$. The IVFN-SHS values for $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4$ and ϑ_5 are demonstrated in the following Table 3, Table 4, Table 5, Table 6 and Table 7.

Table 3
IVFN-SHS values for Organ compatibility

Alternatives	δ_1	δ_2	δ_3	δ_4
A_1	$\begin{pmatrix}(0.48, 0.57),\\(0.47, 0.35),\\(0.62, 0.47)\end{pmatrix}$	$\begin{pmatrix}(0.74, 0.89),\\(0.38, 0.36),\\(0.29, 0.21)\end{pmatrix}$	$\begin{pmatrix}(0.17, 0.26),\\(0.38, 0.50),\\(0.48, 0.77)\end{pmatrix}$	$\begin{pmatrix}(0.28, 0.51),\\(0.34, 0.36),\\(0.36, 0.90)\end{pmatrix}$
A_2	$\begin{pmatrix}(0.25, 0.48),\\(0.42, 0.83),\\(0.62, 0.58)\end{pmatrix}$	$\begin{pmatrix}(0.11, 0.29),\\(0.40, 0.63),\\(0.37, 0.29)\end{pmatrix}$	$\begin{pmatrix}(0.69, 0.95),\\(0.56, 0.48),\\(0.36, 0.23)\end{pmatrix}$	$\begin{pmatrix}(0.88, 0.01),\\(0.36, 0.99),\\(0.58, 0.74)\end{pmatrix}$
A_3	$\begin{pmatrix}(0.34, 0.25),\\(0.14, 0.68),\\(0.12, 0.19)\end{pmatrix}$	$\begin{pmatrix}(0.35, 0.53),\\(0.93, 0.48),\\(0.21, 0.49)\end{pmatrix}$	$\begin{pmatrix}(0.92, 0.73),\\(0.28, 0.39),\\(0.37, 0.62)\end{pmatrix}$	$\begin{pmatrix}(0.95, 0.77),\\(0.67, 0.25),\\(0.15, 0.36)\end{pmatrix}$

Table 4
IVFN-SHS values for Urgency Level

Alternatives	δ_1	δ_2	δ_3	δ_4
B_1	$\begin{pmatrix}((0.14, 0.73),\\(0.28, 0.42),\\(0.19, 0.91))\end{pmatrix}$	$\begin{pmatrix}((0.41, 0.02),\\(0.76, 0.16),\\(0.53, 0.11))\end{pmatrix}$	$\begin{pmatrix}((0.43, 0.20),\\(0.66, 0.77),\\(0.10, 0.62))\end{pmatrix}$	$\begin{pmatrix}((0.23, 0.40),\\(0.55, 0.87),\\(0.33, 0.52))\end{pmatrix}$
B_2	$\begin{pmatrix}((0.67, 0.85),\\(0.30, 0.32),\\(0.88, 0.56))\end{pmatrix}$	$\begin{pmatrix}((0.29, 0.82),\\(0.45, 0.68),\\(0.39, 0.94))\end{pmatrix}$	$\begin{pmatrix}((0.90, 0.37),\\(0.16, 0.59),\\(0.81, 0.44))\end{pmatrix}$	$\begin{pmatrix}((0.99, 0.18),\\(0.75, 0.26),\\(0.12, 0.60))\end{pmatrix}$
B_3	$\begin{pmatrix}((0.13, 0.98),\\(0.79, 0.49),\\(0.24, 0.61))\end{pmatrix}$	$\begin{pmatrix}((0.58, 0.31),\\(0.22, 0.80),\\(0.65, 0.17))\end{pmatrix}$	$\begin{pmatrix}((0.27, 0.64),\\(0.72, 0.15),\\(0.89, 0.46))\end{pmatrix}$	$\begin{pmatrix}((0.97, 0.25),\\(0.83, 0.70),\\(0.48, 0.36))\end{pmatrix}$

IVFN-SHS is given by, $\mathfrak{W} : \vartheta_1 \times \vartheta_2 \times \vartheta_3 \times \vartheta_4 \times \vartheta_5 \rightarrow \vartheta_n(\mathfrak{X})$. Consider that $\mathfrak{W}$(T) = $\mathfrak{W}$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\delta_2, \delta_3\}$ as shown in the Table 8. Then IVFN-SHSS of this relation is given by,
$\mathfrak{W}$(T) = $\mathfrak{W}$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\{\delta_2$,(High Urgency level (F_1) $\{$(0.58,0.31),(0.22,0.80),(0.65,0.17)$\}$, High Health condition of patient (F_2) $\{$(0.75,0.32),(0.48,0.19),(0.61,0.20)$\}$, High Expected quality of patient's life after transplantation (F_3) $\{$(0.13,0.45),(0.22,0.41),(0.85,0.98)$\}$)$\}$,

Table 5
IVFN-SHS values for Age

Age	δ_1	δ_2	δ_3	δ_4
C_1	$\begin{pmatrix}((0.93,0.51),\\(0.57,0.30),\\(0.74,0.86))\end{pmatrix}$	$\begin{pmatrix}((0.47,0.01),\\(0.06,0.78),\\(0.53,0.63))\end{pmatrix}$	$\begin{pmatrix}((0.23,0.87),\\(0.26,0.52),\\(0.83,0.12))\end{pmatrix}$	$\begin{pmatrix}((0.91,0.19),\\(0.56,0.67),\\(0.32,0.85))\end{pmatrix}$
C_2	$\begin{pmatrix}((0.34,0.21),\\(0.63,0.95),\\(0.08,0.50))\end{pmatrix}$	$\begin{pmatrix}((0.39,0.05),\\(0.49,0.90),\\(0.52,0.04))\end{pmatrix}$	$\begin{pmatrix}((0.16,0.97),\\(0.38,0.50),\\(0.69,0.56))\end{pmatrix}$	$\begin{pmatrix}((0.88,0.53),\\(0.39,0.76),\\(0.45,0.68))\end{pmatrix}$
C_3	$\begin{pmatrix}((0.71,0.37),\\(0.54,0.92),\\(0.69,0.84))\end{pmatrix}$	$\begin{pmatrix}((0.95,0.86),\\(0.04,0.74),\\(0.66,0.77))\end{pmatrix}$	$\begin{pmatrix}((0.53,0.11),\\(0.79,0.35),\\(0.85,0.73))\end{pmatrix}$	$\begin{pmatrix}((0.39,0.64),\\(0.73,0.69),\\(0.81,0.91))\end{pmatrix}$

Table 6
IVFN-SHS Values for Health Condition of Patient

Alternatives	δ_1	δ_2	δ_3	δ_4
D_1	$\begin{pmatrix}((0.01,0.85),\\(0.98,0.31),\\(0.41,0.02))\end{pmatrix}$	$\begin{pmatrix}((0.03,0.13),\\(0.04,0.79),\\(0.62,0.25))\end{pmatrix}$	$\begin{pmatrix}((0.25,0.49),\\(0.06,0.88),\\(0.71,0.36))\end{pmatrix}$	$\begin{pmatrix}((0.28,0.76),\\(0.29,0.53),\\(0.08,0.94))\end{pmatrix}$
D_2	$\begin{pmatrix}((0.29,0.56),\\(0.67,0.15),\\(0.44,0.61))\end{pmatrix}$	$\begin{pmatrix}((0.41,0.02),\\(0.82,0.12),\\(0.13,0.68))\end{pmatrix}$	$\begin{pmatrix}((0.76,0.11),\\(0.93,0.58),\\(0.12,0.35))\end{pmatrix}$	$\begin{pmatrix}((0.99,0.17),\\(0.65,0.42),\\(0.19,0.10))\end{pmatrix}$
D_3	$\begin{pmatrix}((0.90,0.23),\\(0.11,0.17),\\(0.84,0.18))\end{pmatrix}$	$\begin{pmatrix}((0.75,0.32),\\(0.48,0.19),\\(0.61,0.20))\end{pmatrix}$	$\begin{pmatrix}((0.97,0.21),\\(0.50,0.38),\\(0.24,0.59))\end{pmatrix}$	$\begin{pmatrix}((0.22,0.40),\\(0.15,0.70),\\(0.23,0.43))\end{pmatrix}$

Table 7
IVFN-SHS Values for Expected Quality of Patient's Life After Transplantation

Alternatives	δ_1	δ_2	δ_3	δ_4
E_1	$\begin{pmatrix}((0.86,0.24),\\(0.80,0.52),\\(0.55,0.27))\end{pmatrix}$	$\begin{pmatrix}((0.33,0.62),\\(0.09,0.66),\\(0.27,0.55))\end{pmatrix}$	$\begin{pmatrix}((0.73,0.82),\\(0.18,0.95),\\(0.92,0.63))\end{pmatrix}$	$\begin{pmatrix}((0.40,0.03),\\(0.20,0.83),\\(0.13,0.77))\end{pmatrix}$
E_2	$\begin{pmatrix}((0.44,0.32),\\(0.39,0.92),\\(0.33,0.87))\end{pmatrix}$	$\begin{pmatrix}((0.30,0.34),\\(0.54,0.69),\\(0.35,0.16))\end{pmatrix}$	$\begin{pmatrix}((0.81,0.36),\\(0.74,0.21),\\(0.37,0.51))\end{pmatrix}$	$\begin{pmatrix}((0.34,0.36),\\(0.60,0.45),\\(0.39,0.08))\end{pmatrix}$
E_3	$\begin{pmatrix}((0.88,0.41),\\(0.32,0.57),\\(0.96,0.68))\end{pmatrix}$	$\begin{pmatrix}((0.13,0.45),\\(0.22,0.41),\\(0.85,0.98))\end{pmatrix}$	$\begin{pmatrix}((0.79,0.62),\\(0.49,0.53),\\(0.28,0.62))\end{pmatrix}$	$\begin{pmatrix}((0.47,0.94),\\(0.17,0.49),\\(0.50,0.65))\end{pmatrix}$

$\{\delta_3$,(High Urgency level$\{$(0.27,0.64),(0.72,0.15),(0.89,0.46)$\}$,
High Health condition of patient $\{$(0.97,0.21),(0.50,0.38),(0.24,0.59)$\}$, High Expected quality of patient's life after transplantation $\{$(0.79,0.62),(0.49,0.53),(0.28,0.62)$\}$)$\}\}$.

Definition 5: IVFN-SHS Subset

Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the corresponding attribute sets are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j = \phi$, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied, we can say $\mathfrak{W}(T_1)$ is the IVFN-SHS subset of $\mathfrak{W}(T_2)$,

Table 8
IVFN-SHS Values for $\mathfrak{W}$(T)

$\mathfrak{W}(T)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0.58, 0.31),\\(0.22, 0.80),\\(0.65, 0.17)\end{pmatrix}$	$\begin{pmatrix}(0.27, 0.64),\\(0.72, 0.15),\\(0.89, 0.46)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.75, 0.32),\\(0.48, 0.19),\\(0.61, 0.20)\end{pmatrix}$	$\begin{pmatrix}(0.97, 0.21),\\(0.50, 0.38),\\(0.24, 0.59)\end{pmatrix}$
F_3	$\begin{pmatrix}(0.13, 0.45),\\(0.22, 0.41),\\(0.85, 0.98)\end{pmatrix}$	$\begin{pmatrix}(0.79, 0.62),\\(0.49, 0.53),\\(0.28, 0.62)\end{pmatrix}$

$$T^M(\mathfrak{W}(T_1)) \leq T^M(\mathfrak{W}(T_2)), T^N(\mathfrak{W}(T_1)) \leq T^M(\mathfrak{W}(T_2)),$$
$$I^M(\mathfrak{W}(T_1)) \geq I^M(\mathfrak{W}(T_2)), I^N(\mathfrak{W}(T_1)) \geq I^M(\mathfrak{W}(T_2)),$$
$$F^M(\mathfrak{W}(T_1)) \geq F^M(\mathfrak{W}(T_2)), F^N(\mathfrak{W}(T_1)) \geq F^M(\mathfrak{W}(T_2)).$$

Example 2:
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$. Then IVFN-SHS $\mathfrak{W}(T_1)$ = $\mathfrak{W}(T_1)$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\delta_2, \delta_3\}$ is the subset of $\mathfrak{W}(T_2)$ = $\mathfrak{W}$(High Urgency level, High Health condition of patient) = $\{\delta_2\}$ as shown in the Table 9 and Table 10, respectively.

Table 9
IVFN-SHS values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0.58, 0.31),\\(0.22, 0.80),\\(0.65, 0.17)\end{pmatrix}$	$\begin{pmatrix}(0.27, 0.64),\\(0.72, 0.15),\\(0.89, 0.46)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.75, 0.32),\\(0.48, 0.19),\\(0.61, 0.20)\end{pmatrix}$	$\begin{pmatrix}(0.97, 0.21),\\(0.50, 0.38),\\(0.24, 0.59)\end{pmatrix}$
F_3	$\begin{pmatrix}(0.13, 0.45),\\(0.22, 0.41),\\(0.85, 0.98)\end{pmatrix}$	$\begin{pmatrix}(0.79, 0.62),\\(0.49, 0.53),\\(0.28, 0.62)\end{pmatrix}$

Definition 6: IVFN-SHS Equality
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the corresponding attribute sets are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j$ = ϕ, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied, we can say $\mathfrak{W}(T_1)$ is equal to $\mathfrak{W}(T_2)$,

$$T^M(\mathfrak{W}(T_1)) = T^M(\mathfrak{W}(T_2)), T^N(\mathfrak{W}(T_1)) = T^M(\mathfrak{W}(T_2)),$$
$$I^M(\mathfrak{W}(T_1)) = I^M(\mathfrak{W}(T_2)), I^N(\mathfrak{W}(T_1)) = I^M(\mathfrak{W}(T_2)),$$
$$F^M(\mathfrak{W}(T_1)) = F^M(\mathfrak{W}(T_2)), F^N(\mathfrak{W}(T_1)) = F^M(\mathfrak{W}(T_2)).$$

Table 10
IVFN-SHS Values for $\mathfrak{W}(T_2)$.

$\mathfrak{W}(T_2)$	δ_2
F_1	$\begin{pmatrix}(0.68,0.37),\\(0.15,0.71),\\(0.54,0.07)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.95,0.62),\\(0.23,0.11),\\(0.41,0.02)\end{pmatrix}$

Example 3:
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$. Then IVFN-SHS $\mathfrak{W}(T_1)$ = $\mathfrak{W}(T_1)$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\delta_2, \delta_3\}$ is equal to $\mathfrak{W}(T_2)$ = $\mathfrak{W}$(High Urgency level, High Health condition of patient) = $\{\delta_2\}$ as shown in the Table 11 and Table 12, respectively.

Table 11
IVFN-SHS Values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0.58,0.31),\\(0.22,0.80),\\(0.65,0.17)\end{pmatrix}$	$\begin{pmatrix}(0.27,0.64),\\(0.72,0.15),\\(0.89,0.46)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.75,0.32),\\(0.48,0.19),\\(0.61,0.20)\end{pmatrix}$	$\begin{pmatrix}(0.97,0.21),\\(0.50,0.38),\\(0.24,0.59)\end{pmatrix}$
F_3	$\begin{pmatrix}(0.13,0.45),\\(0.22,0.41),\\(0.85,0.98)\end{pmatrix}$	$\begin{pmatrix}(0.79,0.62),\\(0.49,0.53),\\(0.28,0.62)\end{pmatrix}$

Table 12
IVFN-SHS Values for $\mathfrak{W}(T_2)$.

$\mathfrak{W}(T_2)$	δ_2
F_1	$\begin{pmatrix}(0.58,0.31),\\(0.22,0.80),\\(0.65,0.17)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.75,0.32),\\(0.48,0.19),\\(0.61,0.20)\end{pmatrix}$

Definition 7: IVFN-SHS Null Set
Suppose that we have IVFN-SHS sets namely $\mathfrak{W}(T_1)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ as shown in the Table 13, and set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the sets of corresponding attributes are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j$ = ϕ, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied,

we can say $\mathfrak{W}(T_1)$ is IVFN-SHS Null set,

$$T^M(\mathfrak{W}(T_1)) = 0, T^N(\mathfrak{W}(T_1)) = 0,$$
$$I^M(\mathfrak{W}(T_1)) = 0, I^N(\mathfrak{W}(T_1)) = 0,$$
$$F^M(\mathfrak{W}(T_1)) = 0, F^N(\mathfrak{W}(T_1)) = 0.$$

Example 4: Suppose that we have an IVFN-SHS set, namely $\mathfrak{W}(T_1)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$. Then IVFN-SHS $\mathfrak{W}(T_1)$ = $\mathfrak{W}(T_1)$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\delta_2, \delta_3\}$ is Null set as shown in the Table 13.

Table 13
IVFN-SHS Values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$
F_2	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$
F_3	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$

Definition 8: IVFN-SHS Compliment
Suppose that we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and the set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the sets of corresponding attributes are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j$ = ϕ, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied, we can say that union between $(\mathfrak{W}(T_1))$ and $(\mathfrak{W}(T_2))$ is the compliment of $\mathfrak{W}(T_1)$,

$$T^M(\mathfrak{W}(T_1)) = F^M(\mathfrak{W}(T_1)), T^N(\mathfrak{W}(T_1)) = F^N(\mathfrak{W}(T_1)),$$
$$I^M(\mathfrak{W}(T_1)) = 1 - I^N(\mathfrak{W}(T_1)), I^N(\mathfrak{W}(T_1)) = 1 - I^M(\mathfrak{W}(T_1)),$$
$$F^M(\mathfrak{W}(T_1)) = T^M(\mathfrak{W}(T_1)), F^N(\mathfrak{W}(T_1)) = T^N(\mathfrak{W}(T_1)).$$

Example 5:
Suppose we have a IVFN-SHS set namely $\mathfrak{W}(T_1)$(High Urgency level, High Health condition of patient, High Expected quality of patient's life after transplantation) = $\{\delta_2, \delta_3\}$. Then IVFN-SHS $\mathfrak{W}(T_1)$ has the compliment $(\mathfrak{W}(T_1))^C$ as shown in the Table 14 and Table 15, respectively.
Definition 9: IVFN-SHS Union
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the corresponding attribute sets are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j$ = ϕ, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied, we can say that union between $(\mathfrak{W}(T_1))$ and $(\mathfrak{W}(T_2))$ is given by $(\mathfrak{W}(T_1)) \cup (\mathfrak{W}(T_2))$,

Table 14
IVFN-SHS Values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0.58, 0.31),\\(0.22, 0.80),\\(0.65, 0.17)\end{pmatrix}$	$\begin{pmatrix}(0.27, 0.64),\\(0.72, 0.15),\\(0.89, 0.46)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.75, 0.32),\\(0.48, 0.19),\\(0.61, 0.20)\end{pmatrix}$	$\begin{pmatrix}(0.97, 0.21),\\(0.50, 0.38),\\(0.24, 0.59)\end{pmatrix}$
F_3	$\begin{pmatrix}(0.13, 0.45),\\(0.22, 0.41),\\(0.85, 0.98)\end{pmatrix}$	$\begin{pmatrix}(0.79, 0.62),\\(0.49, 0.53),\\(0.28, 0.62)\end{pmatrix}$

Table 15
IVFN-SHS values for $(\mathfrak{W}(T_1))^C$.

$\mathfrak{W}(T_1)$	δ_2	δ_3
F_1	$\begin{pmatrix}(0.65, 0.17),\\(0.20, 0.78),\\(0.58, 0.31)\end{pmatrix}$	$\begin{pmatrix}(0.89, 0.46),\\(0.85, 0.28),\\(0.27, 0.64)\end{pmatrix}$
F_2	$\begin{pmatrix}(0.61, 0.20),\\(0.19, 0.48),\\(0.75, 0.32)\end{pmatrix}$	$\begin{pmatrix}(0.24, 0.59),\\(0.62, 0.50),\\(0.97, 0.21)\end{pmatrix}$
F_3	$\begin{pmatrix}(0.85, 0.98),\\(0.59, 0.78),\\(0.13, 0.45)\end{pmatrix}$	$\begin{pmatrix}(0.28, 0.62),\\(0.47, 0.51),\\(0.79, 0.62)\end{pmatrix}$

$$(\mathfrak{W}(T_1)) \cup (\mathfrak{W}(T_2)) =$$
$$\{[T^M(\mathfrak{W}(T)), T^N(\mathfrak{W}(T))], [I^M(\mathfrak{W}(T)), I^N(\mathfrak{W}(T))], [F^M(\mathfrak{W}(T)), F^N(\mathfrak{W}(T))]\}$$
$$T^M(\mathfrak{W}(T)) = max\{T^M(\mathfrak{W}(T_1)), T^M(\mathfrak{W}(T_2))\}, T^N(\mathfrak{W}(T)) = max\{T^N(\mathfrak{W}(T_1)), T^N(\mathfrak{W}(T_2))\},$$
$$I^M(\mathfrak{W}(T)) = min\{I^M(\mathfrak{W}(T_1)), I^M(\mathfrak{W}(T_2))\}, I^N(\mathfrak{W}(T)) = min\{I^N(\mathfrak{W}(T_1)), I^N(\mathfrak{W}(T_2))\},$$
$$F^M(\mathfrak{W}(T)) = min\{F^M(\mathfrak{W}(T_1)), F^M(\mathfrak{W}(T_2))\}, F^N(\mathfrak{W}(T)) = min\{F^N(\mathfrak{W}(T_1)), F^N(\mathfrak{W}(T_2))\},$$

Example 6:
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ as shown in the Table 16 and Table 17, respectively. Then the union between $\mathfrak{W}(T_1)$ = $\mathfrak{W}(T_1)$((Medium Organ compatibility G_1, Medium Urgency level G_2, Medium Age G_3, Medium Health condition of patient G_4) = $\{\delta_1, \delta_3\}$ and $\mathfrak{W}(T_2)$ = $\mathfrak{W}$(Medium Organ compatibility , Medium Age, Medium Health condition of patient) = $\{\delta_1\}$ is shown in the Table 18 and given by.

Definition 10: IVFN-SHS Intersection
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and set of attributes is "A = $\{\alpha_1, \alpha_2, \alpha_3, .., \alpha_n\}$" for n $\geq$ 1 related to "$\mathfrak{X}$". In addition, the corresponding attribute sets are $\beta_1, \beta_2, \beta_3, ..., \beta_n$ with $\beta_i \cap \beta_j = \phi$, for i,j $\in$ 1,2,3,...,n and i $\neq$ j. Consider that the power sets of $\beta_1, \beta_2, \beta_3, ..., \beta_n$ are given by $\mathcal{P}(\beta_1), \mathcal{P}(\beta_2), \mathcal{P}(\beta_3), ..., \mathcal{P}(\beta_n)$ respectively. Also, the relation defined $\mathcal{P}(\beta_1) \times \mathcal{P}(\beta_2) \times \mathcal{P}(\beta_3) \times ... \times \mathcal{P}(\beta_n) = T$. If the following conditions are satisfied, we can say that union between $(\mathfrak{W}(T_1))$ and $(\mathfrak{W}(T_2))$ is given by $(\mathfrak{W}(T_1)) \cap (\mathfrak{W}(T_2))$,

Table 16
IVFN-SHS Values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_1	δ_3
G_1	$\begin{pmatrix}(0.25,0.48),\\(0.42,0.83),\\(0.62,0.58)\end{pmatrix}$	$\begin{pmatrix}(0.69,0.95),\\(0.56,0.48),\\(0.36,0.23)\end{pmatrix}$
G_2	$\begin{pmatrix}(0.67,0.85),\\(0.30,0.32),\\(0.88,0.56)\end{pmatrix}$	$\begin{pmatrix}(0.90,0.37),\\(0.16,0.59),\\(0.81,0.44)\end{pmatrix}$
G_3	$\begin{pmatrix}(0.34,0.21),\\(0.63,0.95),\\(0.08,0.50)\end{pmatrix}$	$\begin{pmatrix}(0.16,0.97),\\(0.38,0.50),\\(0.69,0.56)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.29,0.56),\\(0.67,0.15),\\(0.44,0.61)\end{pmatrix}$	$\begin{pmatrix}(0.76,0.11),\\(0.93,0.58),\\(0.12,0.35)\end{pmatrix}$

Table 17
IVFN-SHS values for $\mathfrak{W}(T_2)$.

$\mathfrak{W}(T_1)$	δ_1
G_1	$\begin{pmatrix}(0.11,0.29),\\(0.40,0.63),\\(0.37,0.29)\end{pmatrix}$
G_2	$\begin{pmatrix}(0.49,0.25),\\(0.42,0.98),\\(0.52,0.34)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.43,0.92),\\(0.12,0.12),\\(0.43,0.78)\end{pmatrix}$

Table 18
IVFN-SHS values for $\mathfrak{W}(T_1) \cup \mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1) \cup \mathfrak{W}(T_2)$	δ_1	δ_3
G_1	$\begin{pmatrix}(0.25,0.48),\\(0.40,0.63),\\(0.37,0.29)\end{pmatrix}$	$\begin{pmatrix}(0.69,0.95),\\(0.56,0.48),\\(0.36,0.23)\end{pmatrix}$
G_2	$\begin{pmatrix}(0.67,0.85),\\(0.30,0.32),\\(0.88,0.56)\end{pmatrix}$	$\begin{pmatrix}(0.90,0.37),\\(0.16,0.59),\\(0.81,0.44)\end{pmatrix}$
G_3	$\begin{pmatrix}(0.49,0.25),\\(0.42,0.95),\\(0.08,0.34)\end{pmatrix}$	$\begin{pmatrix}(0.16,0.97),\\(0.38,0.50),\\(0.69,0.56)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.43,0.92),\\(0.12,0.12),\\(0.43,0.61)\end{pmatrix}$	$\begin{pmatrix}(0.76,0.11),\\(0.93,0.58),\\(0.12,0.35)\end{pmatrix}$

$$(\mathfrak{W}(T_1)) \cap (\mathfrak{W}(T_2)) =$$
$$\{[T^M(\mathfrak{W}(T)), T^N(\mathfrak{W}(T))], [I^M(\mathfrak{W}(T)), I^N(\mathfrak{W}(T))], [F^M(\mathfrak{W}(T)), F^N(\mathfrak{W}(T))]\}$$
$$T^M(\mathfrak{W}(T)) = min\{T^M(\mathfrak{W}(T_1)), T^M(\mathfrak{W}(T_2))\}, T^N(\mathfrak{W}(T)) = min\{T^N(\mathfrak{W}(T_1)), T^N(\mathfrak{W}(T_2))\},$$
$$I^M(\mathfrak{W}(T)) = max\{I^M(\mathfrak{W}(T_1)), I^M(\mathfrak{W}(T_2))\}, I^N(\mathfrak{W}(T)) = max\{I^N(\mathfrak{W}(T_1)), I^N(\mathfrak{W}(T_2))\},$$
$$F^M(\mathfrak{W}(T)) = max\{F^M(\mathfrak{W}(T_1)), F^M(\mathfrak{W}(T_2))\}, F^N(\mathfrak{W}(T)) = max\{F^N(\mathfrak{W}(T_1)), F^N(\mathfrak{W}(T_2))\},$$

Example 7:
Suppose we have two IVFN-SHS sets namely $\mathfrak{W}(T_1)$ and $\mathfrak{W}(T_2)$ over $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ as shown in the Table 19 and Table 20, respectively. Then the union between $\mathfrak{W}(T_1)$ = $\mathfrak{W}(T_1)$((Medium Organ compatibility , Medium Urgency level, Medium Age, Medium Health condition of patient)) = $\{\delta_1, \delta_3\}$ and $\mathfrak{W}(T_2)$ = $\mathfrak{W}$(Medium Organ compatibility , Medium Age, Medium Health condition of patient) = $\{\delta_1\}$ is shown in the Table 21 and given by.

Table 19
IVFN-SHS values for $\mathfrak{W}(T_1)$.

$\mathfrak{W}(T_1)$	δ_1	δ_3
G_1	$\begin{pmatrix}(0.25, 0.48),\\(0.42, 0.83),\\(0.62, 0.58)\end{pmatrix}$	$\begin{pmatrix}(0.69, 0.95),\\(0.56, 0.48),\\(0.36, 0.23)\end{pmatrix}$
G_2	$\begin{pmatrix}(0.67, 0.85),\\(0.30, 0.32),\\(0.88, 0.56)\end{pmatrix}$	$\begin{pmatrix}(0.90, 0.37),\\(0.16, 0.59),\\(0.81, 0.44)\end{pmatrix}$
G_3	$\begin{pmatrix}(0.34, 0.21),\\(0.63, 0.95),\\(0.08, 0.50)\end{pmatrix}$	$\begin{pmatrix}(0.16, 0.97),\\(0.38, 0.50),\\(0.69, 0.56)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.29, 0.56),\\(0.67, 0.15),\\(0.44, 0.61)\end{pmatrix}$	$\begin{pmatrix}(0.76, 0.11),\\(0.93, 0.58),\\(0.12, 0.35)\end{pmatrix}$

Table 20
IVFN-SHS values for $\mathfrak{W}(T_2)$.

$\mathfrak{W}(T_2)$	δ_1
G_1	$\begin{pmatrix}(0.11, 0.29),\\(0.40, 0.63),\\(0.37, 0.29)\end{pmatrix}$
G_2	$\begin{pmatrix}(0.49, 0.25),\\(0.42, 0.98),\\(0.52, 0.34)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.43, 0.92),\\(0.12, 0.12),\\(0.43, 0.78)\end{pmatrix}$

Table 21
IVFN-SHS values for $\mathfrak{W}(T_1) \cap \mathfrak{W}(T_2)$.

$\mathfrak{W}(T_1) \cap \mathfrak{W}(T_1)$	δ_1
G_1	$\begin{pmatrix}(0.11, 0.29),\\(0.42, 0.83),\\(0.62, 0.58)\end{pmatrix}$
G_2	$\begin{pmatrix}(0, 0),\\(0.30, 0.32),\\(0.88, 0.56)\end{pmatrix}$
G_3	$\begin{pmatrix}(0.34, 0.21),\\(0.63, 0.98),\\(0.52, 0.50)\end{pmatrix}$
G_4	$\begin{pmatrix}(0.29, 0.56),\\(0.67, 0.15),\\(0.44, 0.78)\end{pmatrix}$

5. Conclusions

Overall, our research contributes to the field of decision-making by tackling complex scenarios with multiple qualities and ambiguity. Hypersoft sets, especially when mixed with interval-valued Fermatean Neutrosophic sets, make it simple to solve problems that have more than one attribute. Incorporating new algebraic operations, such as subset, equality, null set, complement, union, intersection, and decision-making applications for prioritizing organ transplant patients based on criteria like organ compatibility, age, health condition, and urgency level, enhances the model's analytical capabilities. The interval-valued Fermatean Neutrosophic Super HyperSoft Sets are used in this paper to come up with definitions and mathematical operations that can be used to solve real-world decision-making problems. The numerous suggestions made in this paper could greatly benefit future research in this developing area. It is worthwhile to look into hybrid models that combine several decision-making approaches, modify the framework to handle different kinds of data and uncertainties, and improve the computing performance. Studying the DM process without considering the interdependencies, dependencies, or interdependencies between the qualities could lead to confusion. In the future, researchers may use machine learning tools like regression to better understand and measure these connections. The area could advance in the future as researchers investigate more complex fuzzy collections. Researchers can also use the same approach in other areas related to electric vehicles, decision trees, type-2 networks, signal decomposition, predictive control models, etc. The suggested approach is flexible enough to be modified for use in feature selection and other machine learning tasks.

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Conflicts of Interest

The authors declare no conflict of interest.

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