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Quantifying Multi-Cause Psychological Disorder Risk Through an Advanced Mathematical Model Using Intuitionistic Pentagonal Fuzzy Logic

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ABSTRACT

The evaluation of psychological disorders is a very complicated and sensitive area of study, where emotions, thoughts, and the mental states of different patients are analyzed on the basis of questionnaires and observational techniques. The identification and analysis of people's mental states, emotions, and behaviors are difficult and limited with the aid of conventional approaches since the data gathered from these factors appear to be vague, uncertain, and imprecise. To demonstrate this information systematically, we have proposed a comprehensive mathematical model able to deal with indeterminate and imprecise data, utilizing Intuitionistic Pentagonal Fuzzy Numbers (IPnFNs). To create a coherent mathematical framework, IPnFNs integrate five values of membership and non-membership functions of any constraint, providing an innovative numerical framework that effectively explains the assessment of diagnosis and the patient's condition. The research study analyzes several key psychiatric disorders, such as childhood psychiatric problems, hormonal changes, social stress, cognitive distortions, anxiety, and traumatic life events, as the main factors. The proposed technique provides psychologists with a thorough and clear picture of the issue by depicting the patient's mental health numerically. The research is significantly helpful in cases where the symptoms of psychological illness are not obvious or the patient's information is imprecise or confusing. Furthermore, a comparative analysis of various patients can be developed using the proposed technique, assisting in assessing severity and diagnosis preferences. This mathematical method shows that the use of IPnFNs can be a reliable and scientific method in the evaluation of psychological disorders. This approach is significantly applicable in educational counseling and clinical psychology, where it is a daily routine for individuals to deal with imprecise and inconsistent data in mental health issues while making decisions.

Keywords:
Multi-Criteria Decision Making; Intuitionistic Pentagonal Fuzzy Framework; Fuzzy Logical operators; Psychiatric Disorder Evaluation

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1. Introduction

The human mind is a complicated network that handles all types of behaviors, emotions, thoughts, and decision-making processes. Between the structure and performance of the human brain, there is an equilibrium, but several internal and external key factors affect this equilibrium [1, 2]. Psychological problems such as depression, mental stress, anxiety, or several psychological disorders happen when this equilibrium is abrupt, these issues are referred to as "mental illnesses" or "psychiatric disorders" [3]. These mental illnesses include bipolar disorder, OCD, multi-personality disorders, schizophrenia, anxiety, and stress. In this era, the challenges of mental health have become not only an individual problem but a global and social challenge [4, 5]. A study held by the World Health Organization shows that every fourth person is acquainted with some kind of psychiatric disorder sooner or later in their lives [6]. Unlike physical diseases, psychological disorders appear without any visible or apparent symptoms, so their identification and classification are complicated and sensitive procedures; therefore diagnosis of mental disorders has always been a provocation [7]. Even in this modern age, psychologists often utilize traditional methods like questionnaires and observational evaluations on the basis of patients' speech, abrupt behavior, body language, attitudes, and mood swings. Even though these techniques have revolutionized however they largely rely on patient self-evaluation and subjective feedback, but a quantitative and definitive basis cannot be developed [8]. Considering this basic problem, the researchers resort to developing innovative mathematical and scientific techniques to demonstrate complex, ambiguous, and non-standard conceptions like human emotions and thoughts with mathematical tools. Modern approaches such as Fuzzy Theory, Neutrosophic Systems, and Intuitionistic Fuzzy Logic have made notable progress in this area of study [9–11].

Fuzzy theory was developed by Lofti A. Zadeh in 1965, and is a mathematical study introduced to acknowledge and demonstrate these kinds of phenomena where the data is imprecise, unclear, or incomplete [12]. So, uncertain and vague information present in real-world scenarios can be shown as mathematical tools. This method enables us to better describe human behaviors, emotions, attitudes, and other psychological factors because these elements are not simply black or white, but include limitless levels and colors. In 1986, Atanassov developed an advanced theory based on fuzzy logic, including membership and non-membership degrees, namely Intuitionistic Fuzzy Sets (IFS) [13]. Every element of this set is categorized into membership and non-membership grades separately, providing a more flexible and consistent mathematical principle to deal not only with ambiguity and imprecision found in everyday life challenges but also with contradictory information. Afterwards, various revolutionary fuzzy tools were developed like, pythagorean fuzzy sets, fermatean fuzzy sets, neutrosophic sets, spherical fuzzy sets, triangular fuzzy numbers, trapezoidal fuzzy numbers, and many more [14–23]. The pentagonal fuzzy numbers (PnFNs) are characterized into five gradual levels ranging from very low to very high, such that as we move forward, the value gets greater and greater. This fuzzy tool enables a deeper measurement of attitudes, emotions, and human conditions [24]. Integrating intuitionistic fuzzy sets with pentagonal fuzzy numbers yields us a flexible mathematical tool that is intuitionistic pentagonal fuzzy numbers (IPnFNs), which not only aggregates the intensity of the situation but also considers the non-membership and uncertainty factors of it [25]. This trait makes intuitionistic pentagonal fuzzy numbers significantly useful in complicated systems like human psychology because, during the diagnosis of psychiatric disorders, there are often cases where the patient is unable to completely explain the symptoms and causes.

The human brain is a complex web of experiences, emotions, and memories, which cannot be elaborated simply by conventional or verbal questionnaires. While studying the mental illnesses and their effects, we encounter several causes that explain the mental state of the individual, like stress, low self-esteem, lack of sleep, severe sadness, anxiety, suicidal tendencies, disinterest, social isolation, etc. Evaluating these causes only on the basis of a "yes" or "no" questionnaire is an insult to the condition that the patient is going through. In this complicated situation, IPnFNs play a significant role by handling the levels of factors in a more comprehensive way [26]. The proposed technique shows each cause not just in terms of its severity, but it also considers the involvement of uncertainty and non-membership grade. In this mathematical model, causes of psychological disorders are categorized into various key features, social behaviors, internal conflicts, physical patterns, cognitive functions, and emotional symptoms. Every constraint is evaluated with the aid of an IPnFN, and then they are analyzed collectively. Several fuzzy inference rules and logical aggregation techniques have been employed during this procedure. The proposed model not only gives a numerical interpretation but also makes a systematic assessment profile that gives a perceptive overview of the patient's severity of causes, risks, and current condition. This model has allowed the procedure of diagnosis of different psychiatric disorders to be more systematic, scientific, and standardized, also providing various advantages and challenges. A pictorial representation of psychiatric patients is given in the Figure 1.

Fig. 1. Pictorial representation of psychiatric patients.

In mathematical sciences, set theory is a necessary area of study. The theory says that only two outcomes occur when an element appears in a set: either it resides in it or not. This theory is referred to as "Crisp Set," having a membership criterion that can be either 0 or 1. The crisp logic proved to be useful, but it lacked the property to handle factors like imprecision, incompleteness, ambiguity, uncertainty, and inconsistency. To overcome this hurdle, Professor Lofti A. Zadeh gave the idea of "Fuzzy Sets (FS)" in 1965, for a better explanation of the ambiguity and impreciseness present in the current world information [12]. The membership value of any element in a fuzzy set lies in the interval [0,1]. We can also say that an element is present "to some extent" in a collection instead of entirely existing in it or not. In recent times, fuzzy theory and its extensions have revolutionized the mathematical world and become a necessary framework for tackling impreciseness and uncertainty existing in decision making processes, especially in multi-criteria decision making (MCDM), healthcare, engineering, and biomedical sciences. While fuzzy sets are efficient for basic uncertain data, they do not address the non-membership and hesitancy factors of an element. To resolve this problem, Atanassov proposed the idea of "Intuitionistic Fuzzy Sets" (IFS) in 1986, modernizing fuzzy logic by integrating both the membership function and the non-membership function of some variable in the universe [13]. He also explained a hesitancy function for a more flexible and robust way to handle the vague information. The condition for the existence of any element to be in IFS is that the sum of membership and non-membership degrees of an element must not be less than 0 and not be greater than 1. Intuitionistic fuzzy numbers showed much greater flexibility than fuzzy sets, but failed when a broader range of impreciseness and flexibility existed in the data. In an effort to overcome this challenge, Yager introduced Pythagorean Fuzzy Sets (PFS) in 2013, where a certain condition lies on the membership and non-membership degrees of an element, that is, the sum of their squares must not be less than 0 and not be greater than 1 [14]. PFS allowed the researchers to deal with a broader range of impreciseness and ambiguity representation.

Further exploring this theory, Senapati and Yager introduced the concept of Fermatean Fuzzy Sets (FFS), which offered even greater flexibility for highly uncertain and ambiguous information [16]. The condition for the existence of a fermatean fuzzy number is that the sum of cubes of membership and non-membership degrees of an element must not be less than 0 and not be greater than 1. In addition to these set theories, to handle uncertainty and vagueness more effectively, various fuzzy number frameworks have been developed. An advanced fuzzy number characterized by three numerical values that are the lower limit value, middle limit value, and higher limit value, is the most straightforward and fundamental approach, named a triangular fuzzy number (TFN) [20]. TFNs are very efficient in areas where a gradual system of parameters needs to be handled effectively. In spite of all these advantages, triangular fuzzy numbers lack to manage some complicated or imprecise circumstances because they only show one point of perfect precision, despite the fact that these numbers are quick to compute and straightforward. This restriction can be overcome by trapezoidal fuzzy numbers (TrFN), which have been characterized by four different gradual points and have a flat certainty range in the center, extending the realism [21]. In comparison to the triangular fuzzy numbers, trapezoidal fuzzy numbers show typically superior elasticity, but are difficult and complicated to compute, and also these numbers showed significant limitations in handling multi-stage impreciseness and uncertainty.

Throughout time, various types of fuzzy numbers and sets have been introduced to model intricate ambiguity and imprecision in a better manner. Another unique and significant development is the pentagonal fuzzy numbers (PFN), which represent the spread and central tendency of an element by five values that imitate the numerical information [27]. The PFNs show finer granularity and precision, especially in complicated and intricate numerical challenges, compared to triangular or trapezoidal fuzzy numbers, making them efficient for more nuanced decision-making scenarios. Integrating the progressive nature of TFNs and TrFNs, and the vast scope of intuitionistic fuzzy sets, Intuitionistic triangular fuzzy numbers and intuitionistic trapezoidal fuzzy numbers were introduced, respectively [28, 29]. This approach allowed an incorporated way of membership and non-membership functions with gradual dynamics, providing a systematic way and a broad range for handling uncertainty and ambiguity present in real-life scenarios. Further exploring intuitionistic fuzzy sets was integrated with pentagonal fuzzy numbers, offering a systematic structure referred to as Intuitionistic Pentagonal Fuzzy Numbers (IPnFNs), to handle the situations where human decisions have various degrees of uncertainty, hesitation, and vagueness [30]. IPnFNs provide improved flexibility, robustness, and accuracy; they are also computationally complex, demanding precise implementation in real-life implications. IPnFNs are really useful in today's uncertain world, handling ambiguity in situations like analyzing financial risk, biomedical engineering, psychological disorder diagnosis, and high-level MCDM [31–34].

In this research study, we have discussed psychological disorders caused by various factors like traumatic life events, genetic factors, social isolation, substance abuse, cognitive decline, hormonal changes, and environmental stressors, etc, in an intuitionistic pentagonal fuzzy environment. We have utilized five-element membership and non-membership degrees of IPnFNs to show the positive and negative variations in the patient's psychological state, providing a systematic way to record even little changes in the patient's mental situation that traditional diagnostics would miss. In comparison with the typical survey data, IPnFN's five-graded system helps to capture the actual condition of a patient's mental state, instead of collecting the response data as "yes" or "no". Instead of using traditional techniques, operators, and methods, we have used a logical method for MCDM, offering a novel approach to handle the uncertainty and vagueness in real-world situations.

Structure of the research article: The structure of this research paper is described as, Section **??** provides the definitions of basic concepts and operations. Section **??** demonstrates about psychological disorders and their various causes and symptoms. Following that, the mathematical formulation of the proposed research study based on the practical example of psychiatric disorder evaluation is given in the Section **??**. To verify our findings, numerical examples have been aggregated in the Section **??**. In the Section **??**, a brief conclusion, some future directions, and limitations of our research work are given. A sequence-wise flow process of the research article is given in the Figure 2.

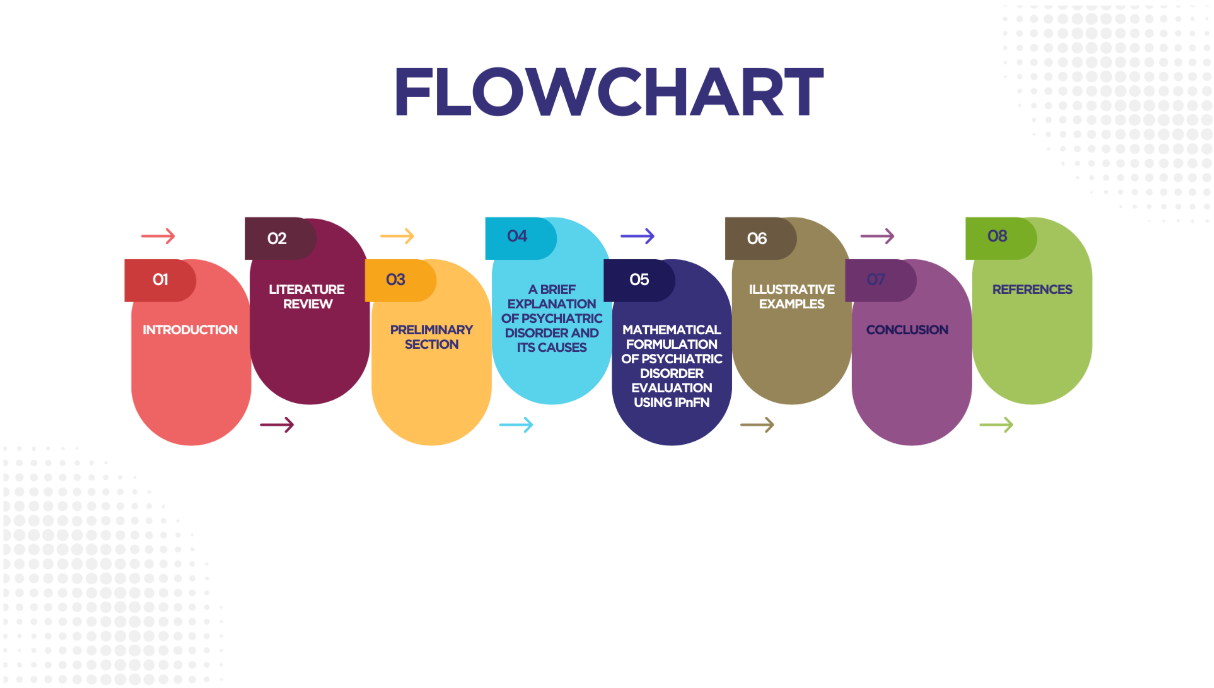

Fig. 2. Flowchart of the research article.

1.1 *Key findings*

Some of the main key findings of our research study are given as follows,

- A richer formulation is provided by IPnFNs for addressing various degrees of vagueness and uncertainty, which conventional fuzzy frameworks fail to capture.
- Several psychological states using parameters like cognition, treatment response, emotional condition, and behavior are evaluated simultaneously, using the pentagonal structure of membership and non-membership of IPnFNs.
- Overlapping symptoms in psychological disorders are efficiently modeled using IPnFNs, unlike traditional approaches like triangular fuzzy numbers or trapezoidal fuzzy numbers.
- Uncertainty in psychological data is better addressed by an IPnFN-based framework, minimizing the risk of misdiagnosis.

- Logistic approach embedded with IPnFN feature offers psychiatrists and clinicians interpretable findings about which set of symptoms affects the clinical predictions the most.
- The proposed technique effectively handles larger data sets of patients, making it capable of handling uncertainty in real-world psychological research studies.

1.2 *Motivation*

In psychology, particularly when working with individuals, we frequently find the notion of clarity to be limited. That's because sometimes patients do not reveal the whole story, plus their emotions act as a fog that make it difficult to determine what is happening. Then, even when the information is somewhat clear, the standard assessments may miss certain things, making it harder to ascertain exactly what the clinical picture looks like and to approach the individual in a treatment process. Therefore, we must identify in-depth pluralized ways of uncertainty, which is systematic and mathematically-rigorous. So, we are using Intuitionistic Pentagonal Fuzzy Numbers (IPnFNs) model. This model offers us a method for understanding and assessing an individual's mental state, that may be more complex. Our aim is to create a tool for psychologists, supporting their decision making with true data, particularly in cases of complex or problematic mental health assessments.

1.3 *Research gaps*

Some significant research gaps that we are going to fill up are as below,

- Psychological assessment tools such as clinical observations, interviews and questionnaires have been around for a long time, however, they do not provide a mathematical structure that could quantify vague or ambiguous answers from the patient. There is still a gap to go in converting qualitative emotional and behavioral data into specific numerical values.
- Fuzzy and probabilistic models existing for mental health do not fully account for the vague nature of psychological information. These models cannot fully exhibit a case when a person is having conflicting feelings or only showing you part of their symptoms. In summary, we don't have methods that can both account for uncertainty, and this unsureness at the same time.
- Many studies have used triangular or trapezoidal fuzzy numbers as a method for assessing mental health. Nevertheless, such methods provide limited variability and can obscure different dimensions of people's feelings. To our knowledge, no one has examined the use of Intuitionistic Pentagonal Fuzzy Numbers (IPnFNs) in this sense, providing the possibility of a more adequate representation of emotions.
- Fuzzy logic and medical decision-making have theoretical models, but there are few studies that take these theoretical models and put them into practice (i.e., in therapy and education). Nonetheless, there remains a shortage of practical models that can be utilized by psychologists in everyday practice for patient diagnosis and evaluation.

2. Preliminary section

Definition 1 *(Fuzzy set [12])*
A membership function is used to specify a fuzzy set by which components of domain X are mapped to a unit interval [0,1]. A fuzzy set $\mathcal{F}$ is defined as following;

$$\mathcal{F} = \{\mathrm{x}, \eta_{\mathcal{F}}(\mathrm{x}); \mathrm{x} \in X\}. \tag{1}$$

Here we define $\eta_{\mathcal{F}} : X \rightarrow [0,1]$ as the membership function of the fuzzy set $\mathcal{F}$ and $\eta_{\mathcal{F}}(\mathrm{x})$ as the membership value of $\mathrm{x} \in X$ in $\mathcal{F}$.

Definition 2 *(Intuitionistic Fuzzy set [13])*
An intuitionistic fuzzy set which we have denoted by $\mathcal{F_I}$ in domain X is defined as,

$$\mathcal{F_I} = \{0 \leq \mu_{F_I}(\mathrm{x}) + \nu_{F_I}(\mathrm{x}) \leq 1; \mathrm{x} \in X\}. \tag{2}$$

Where $\mu_{\mathcal{F_I}}(\mathrm{x})$ and $\nu_{\mathcal{F_I}}(\mathrm{x})$ are the degree of membership function and degree of non-membership function of $\mathrm{x} \in X$ respectively.

Definition 3 *(Triangular Fuzzy number [20])*
We can elaborate the mathematical form of a triangular fuzzy number (TFN) $\mathcal{T}$ as;

$$\mathcal{T} = \{\alpha, \beta, \gamma\}, \tag{3}$$

where α, β, and γ are real numbers, with the condition $\alpha < \beta < \gamma$. We define the membership function of triangular fuzzy number as following;

$$\eta_{\mathcal{T}}(\mathrm{x}) = \begin{Bmatrix} \frac{\mathrm{x}-\alpha}{\beta-\alpha}, & for\ \alpha \leq \mathrm{x} \leq \beta \\ \frac{\gamma-\mathrm{x}}{\gamma-\beta}, & for\ \beta \leq \mathrm{x} \leq \gamma \\ 0, & otherwise \end{Bmatrix}. \tag{4}$$

Definition 4 *(Intuitionistic Triangular Fuzzy Number [29])*
We define an intuitionistic triangular fuzzy number of an intuitionistic fuzzy set $\mathcal{T_I}$ as follows;

$$\mathcal{T_I} = \{(\alpha_1^M, \beta_1^M, \gamma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N)\}. \tag{5}$$

Where $\alpha_1^M, \beta_1^M, \gamma_1^M, \alpha_2^N, \beta_2^N$, and γ_2^N are real numbers and its membership function $\mu_{\mathcal{T_I}}(\mathrm{x})$ and non-membership function $\nu_{\mathcal{T_I}}(\mathrm{x})$ are defined as;

$$\mu_{\mathcal{T_I}}(\mathrm{x}) = \begin{Bmatrix} \frac{\mathrm{x}-\alpha_1^M}{\beta_1^M-\alpha_1^M}, & for\ \alpha_1^M \leq \mathrm{x} \leq \beta_1^M \\ \frac{\gamma_1^M-\mathrm{x}}{\gamma_1^M-\beta_1^M}, & for\ \beta_1^M \leq \mathrm{x} \leq \gamma_1^M \\ 1, & for\ \mathrm{x} = \beta_1^M \\ 0, & Otherwise \end{Bmatrix}, \tag{6}$$

$$\nu_{\mathcal{T_I}}(\mathrm{x}) = \begin{Bmatrix} \frac{\beta_1^M-\mathrm{x}}{\beta_1^M-\alpha_2^N}, & for\ \alpha_2^N \leq \mathrm{x} \leq \beta_1^M \\ \frac{\mathrm{x}-\beta_1^M}{\gamma_2^N-\beta_1^M}, & for\ \beta_1^M \leq \mathrm{x} \leq \gamma_2^N \\ 0, & for\ \mathrm{x} = \beta_1^M \\ 1, & Otherwise \end{Bmatrix}. \tag{7}$$

Definition 5 *(Trapezoidal Fuzzy number [21])*
We can elaborate the mathematical form of a trapezoidal fuzzy number (TrFN) $\mathcal{R}$ as;

$$\mathcal{R} = \{\alpha, \beta, \gamma, \delta\}, \tag{8}$$

where α, β, γ, and δ are real numbers, with the condition $\alpha < \beta < \gamma < \delta$. We define membership function of trapezoidal fuzzy number as following;

$$\eta_{\mathcal{R}}(\mathrm{x}) = \begin{Bmatrix} 0, & for\ \mathrm{x} < \alpha \\ \frac{\mathrm{x}-\alpha}{\beta-\alpha}, & for\ \alpha \leq \mathrm{x} \leq \beta \\ 1, & for\ \beta \leq \mathrm{x} \leq \gamma \\ \frac{\delta-\mathrm{x}}{\delta-\gamma}, & for\ \gamma \leq \mathrm{x} \leq \delta \\ 0, & for\ x > \delta \end{Bmatrix}. \tag{9}$$

Definition 6 *(Intuitionistic Trapezoidal Fuzzy Number [30])*
We define an intuitionistic trapezoidal fuzzy number (ITrFN) of an intuitionistic fuzzy set $\mathcal{R}_{\mathcal{I}}$ as follows;

$$\mathcal{R}_{\mathcal{I}} = \{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N)\}. \tag{10}$$

Where $\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \alpha_2^N, \beta_2^N, \gamma_2^N$, and δ_2^N are real numbers and its membership $\mu_{\mathcal{R}_{\mathcal{I}}}(\mathrm{x})$ and non-membership $\nu_{\mathcal{R}_{\mathcal{I}}}(\mathrm{x})$ functions are defined as;

$$\mu_{\mathcal{R}_{\mathcal{I}}}(\mathrm{x}) = \begin{Bmatrix} \frac{\mathrm{x}-\alpha_1^M}{\beta_1^M-\alpha_1^M}, & for\ \alpha_1^M \le \mathrm{x} \le \beta_1^M \\ 1, & for\ \beta_1^M \le \mathrm{x} \le \gamma_1^M \\ \frac{\delta_1^M-\mathrm{x}}{\delta_1^M-\gamma_1^M}, & for\ \gamma_1^M \le \mathrm{x} \le \delta_1^M \\ 0, & Otherwise \end{Bmatrix}, \tag{11}$$

$$\nu_{\mathcal{T}_{\mathcal{I}}}(\mathrm{x}) = \begin{Bmatrix} \frac{\beta_1^M-\mathrm{x}}{\beta_1^M-\alpha_2^N}, & for\ \alpha_2^N \le \mathrm{x} \le \beta_1^M \\ 0, & for\ \beta_1^M \le \mathrm{x} \le \gamma_1^M \\ \frac{\mathrm{x}-\gamma_1^M}{\delta_2^N-\gamma_1^M}, & for\ \gamma_1^M \le \mathrm{x} \le \delta_2^N \\ 1, & Otherwise \end{Bmatrix}. \tag{12}$$

Definition 7 *(Pentagonal Fuzzy number [28])*
We can elaborate the mathematical form of a Pentagonal fuzzy number (PnFN) $\mathcal{P}$ as;

$$\mathcal{P} = \{\alpha, \beta, \gamma, \delta, \sigma\}, \tag{13}$$

where α, β, γ, δ, and σ are real numbers, with the condition $\alpha < \beta < \gamma < \delta < \sigma$. We define the membership function of pentagonal fuzzy number as following;

$$\eta_{\mathcal{P}}(\mathrm{x}) = \begin{Bmatrix} 0, & for\ \mathrm{x} < \alpha \\ \frac{\mathrm{x}-\alpha}{\beta-\alpha}, & for\ \alpha \le \mathrm{x} \le \beta \\ \frac{\mathrm{x}-\beta}{\gamma-\beta}, & for\ \beta \le \mathrm{x} \le \gamma \\ 1, & for\ \mathrm{x} = \gamma \\ \frac{\delta-\mathrm{x}}{\delta-\gamma}, & for\ \gamma \le \mathrm{x} \le \delta \\ \frac{\sigma-\mathrm{x}}{\sigma-\delta}, & for\ \delta \le \mathrm{x} \le \sigma \\ 0, & for\ x > \sigma \end{Bmatrix}. \tag{14}$$

A pentagonal fuzzy number must satisfy the following conditions;

- *The membership function $\eta_{\mathcal{P}}(\mathrm{x})$ is continuous in [0,1].*
- *The membership function $\eta_{\mathcal{P}}(\mathrm{x})$ is continuous and strictly increasing on $[\alpha, \beta]$ and $[\beta, \gamma]$.*
- *The membership function $\eta_{\mathcal{P}}(\mathrm{x})$ is continuous and strictly decreasing on $[\gamma, \delta]$ and $[\delta, \sigma]$.*

Definition 8 *(Intuitionistic Pentagonal Fuzzy Number [31])*
We define an intuitionistic pentagonal fuzzy number of an intuitionistic fuzzy set $\mathcal{P}_{\mathcal{I}}$ as follows;

$$\mathcal{P}_{\mathcal{I}} = \{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N)\}. \tag{15}$$

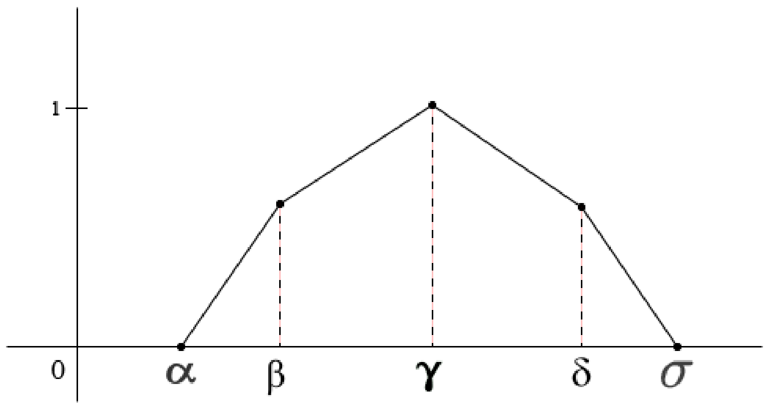

Fig. 3. Graphical form of pentagonal fuzzy number.

Where $\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M, \alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N$, *and* σ_2^N *are real numbers and its membership* $\mu_{\mathcal{P}_{\mathcal{I}}}(\mathrm{x})$ *and non-membership* $\nu_{\mathcal{P}_{\mathcal{I}}}(\mathrm{x})$ *functions are defined as;*

$$\mu_{\mathcal{P}_{\mathcal{I}}}(\mathrm{x}) = \begin{Bmatrix} 0, & \mathrm{x} < \alpha_1^M \\ \frac{\mathrm{x}-\alpha_1^M}{\beta_1^M-\alpha_1^M}, & for\ \alpha_1^M \leq \mathrm{x} \leq \beta_1^M \\ \frac{\mathrm{x}-\beta_1^M}{\gamma_1^M-\beta_1^M}, & for\ \beta_1^M \leq \mathrm{x} \leq \gamma_1^M \\ 1, & for\ \mathrm{x} = \gamma_1^M \\ \frac{\delta_1^M-\mathrm{x}}{\delta_1^M-\gamma_1^M}, & for\ \gamma_1^M \leq \mathrm{x} \leq \delta_1^M \\ \frac{\sigma_1^M-\mathrm{x}}{\sigma_1^M-\delta_1^M}, & for\ \delta_1^M \leq \mathrm{x} \leq \sigma_1^M \\ 0, & x > \sigma_1^M \end{Bmatrix}, \tag{16}$$

$$\nu_{\mathcal{P}_{\mathcal{I}}}(\mathrm{x}) = \begin{Bmatrix} 1, & \mathrm{x} < \alpha_2^N \\ \frac{\beta_2^N-\mathrm{x}}{\beta_2^N-\alpha_2^N}, & for\ \alpha_2^N \leq \mathrm{x} \leq \beta_2^N \\ \frac{\gamma_1^M-\mathrm{x}}{\gamma_1^M-\beta_2^N}, & for\ \beta_2^N \leq \mathrm{x} \leq \gamma_2^N \\ 0, & for\ \mathrm{x} = \gamma_1^M \\ \frac{\mathrm{x}-\gamma_1^M}{\delta_2^N-\gamma_1^M}, & for\ \gamma_2^N \leq \mathrm{x} \leq \delta_2^N \\ \frac{\mathrm{x}-\delta_2^N}{\sigma_2^N-\delta_2^N}, & for\ \delta_2^N \leq \mathrm{x} \leq \sigma_2^N \\ 1, & x > \sigma_2^N \end{Bmatrix}. \tag{17}$$

Definition 9 *(Operations on Intuitionistic Pentagonal Fuzzy Numbers [31])*

- ***Addition of two intuitionistic pentagonal fuzzy numbers:***
  If $\mathcal{P}_{I1} = \left\{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N)\right\}$ *and* $\mathcal{P}_{I2} = \left\{(\alpha_3^M, \beta_3^M, \gamma_3^M, \delta_3^M, \sigma_3^M)(\alpha_4^N, \beta_4^N, \gamma_4^N, \delta_4^N, \sigma_4^N)\right\}$, *are two intuitionistic pentagonal fuzzy numbers then,*

$$\mathcal{P}_{I1} \oplus \mathcal{P}_{I2} = \begin{Bmatrix} \left(\alpha_1^M + \alpha_3^M, \beta_1^M + \beta_3^M, \gamma_1^M + \gamma_3^M, \delta_1^M + \delta_3^M, \sigma_1^M + \sigma_3^M\right), \\ \left(\alpha_2^N + \alpha_4^N, \beta_2^N + \beta_4^N, \gamma_2^N + \gamma_4^N, \delta_2^N + \delta_4^N, \sigma_2^N + \sigma_4^N\right) \end{Bmatrix}. \tag{18}$$

- ***Subtraction of two intuitionistic pentagonal fuzzy numbers:***
  If $\mathcal{P}_{I1} = \left\{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N)\right\}$ *and*

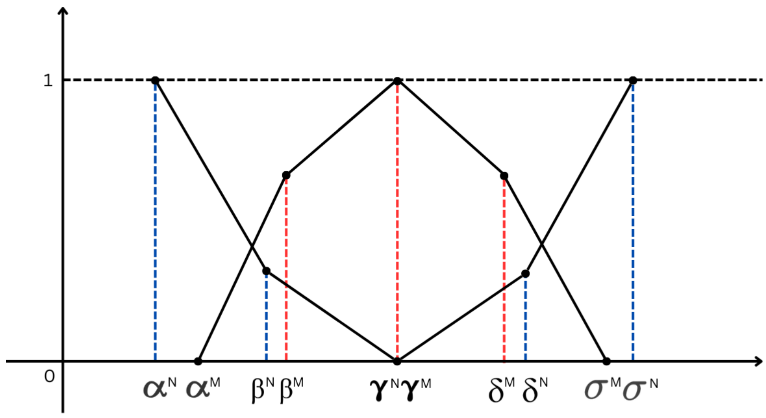

Fig. 4. Graphical form of intuitionistic pentagonal fuzzy number.

$\mathcal{P}_{I2} = \left\{(\alpha_3^M, \beta_3^M, \gamma_3^M, \delta_3^M, \sigma_3^M)(\alpha_4^N, \beta_4^N, \gamma_4^N, \delta_4^N, \sigma_4^N)\right\}$, *are two intuitionistic pentagonal fuzzy numbers then,*

$$\mathcal{P}_{I1} \ominus \mathcal{P}_{I2} = \left\{ \begin{array}{c} \left(\alpha_1^M - \sigma_3^M, \beta_1^M - \delta_3^M, \gamma_1^M - \gamma_3^M, \delta_1^M + \beta_3^M, \sigma_1^M - \alpha_3^M\right), \\ \left(\alpha_2^N - \sigma_4^N, \beta_2^N - \delta_4^N, \gamma_2^N - \gamma_4^N, \delta_2^N - \beta_4^N, \sigma_2^N - \alpha_4^N\right) \end{array} \right\}. \tag{19}$$

Definition 10 *(Score Function of a IPnFN [31])*
We define score function for an intuitionistic pentagonal fuzzy number,
$\mathcal{P}_{\mathcal{I}} = \{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N)\}$ *as;*

$$S(\mathcal{P}_{\mathcal{I}}) = \frac{\{(\alpha_1^M - \alpha_2^N + \beta_1^M - \beta_2^N + \gamma_1^M - \gamma_2^N + \delta_1^M - \delta_2^N + \sigma_1^M - \sigma_1^N)\}}{5}. \tag{20}$$

Definition 11 *(Accuracy Function of an IPnFN [31])*
We define accuracy function for an intuitionistic pentagonal fuzzy number,
$\mathcal{P}_{\mathcal{I}} = \{(\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M)(\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N)\}$ *as;*

$$H(\mathcal{P}_{\mathcal{I}}) = \frac{\{(\alpha_1^M + \alpha_2^N + \beta_1^M + \beta_2^N + \gamma_1^M + \gamma_2^N + \delta_1^M + \delta_2^N + \sigma_1^M + \sigma_1^N)\}}{5}. \tag{21}$$

Definition 12 *(Comparison of score and accuracy function)*

1. *If* $S(\mathcal{P}_{I1}) < S(\mathcal{P}_{I2})$, *then* $\mathcal{P}_{I1} < \mathcal{P}_{I2}$,
2. *If* $S(\mathcal{P}_{I1}) > S(\mathcal{P}_{I2})$, *then* $\mathcal{P}_{I1} > \mathcal{P}_{I2}$,
3. *If* $S(\mathcal{P}_{I1}) = S(\mathcal{P}_{I2})$, *then*
   - *If* $H(\mathcal{P}_{I1}) < H(\mathcal{P}_{I2})$, *then* $\mathcal{P}_{I1} < \mathcal{P}_{I2}$,
   - *If* $H(\mathcal{P}_{I1}) > H(\mathcal{P}_{I2})$, *then* $\mathcal{P}_{I1} > \mathcal{P}_{I2}$,
   - *If* $H(\mathcal{P}_{I1}) = H(\mathcal{P}_{I2})$, *then* $\mathcal{P}_{I1} \approx \mathcal{P}_{I2}$.

3. A brief explanation of psychiatric disorder and its causes

Psychiatric disorder is a matter that has significant effects on human lives, feelings, behavior, minds, education, and relationships. Psychological or psychiatric disorder (PD) can be experienced when there are profound changes in a person's thoughts, emotions, or behavior that influence their daily routine, academics, relationships, and work greatly. There are many causes and effects of psychological disorders in a person. We are going to demonstrate some main causes of this despicable illness and integrate these causes with intuitionistic pentagonal fuzzy numbers (IPnFNs) to make a decision support system that will assist the psychiatrists in numerical aggregating the existence or severity of a PD. Merging the concept of IPnFNs with the evaluation of PD will reduce the uncertainty and vagueness present in these causes. For a better understanding of this ailment, we are going to discuss some main causes of PD, which are given as;

- **Genetic Predisposition:** Psychological disorders often run in families. The chances of having PD in the upcoming generation increase if one of the family members has it(mostly a parent). Mostly, it depends on the genes or gene combinations that reduce the resistance against the PD. For instance, if someone has a close relative suffering from schizophrenia, other family members have a high risk of developing it too.

- **Neurochemical Imbalances:** Anxiety, stress, and depression can cause significant imbalances in neurotransmitters like dopamine, norepinephrine, and serotonin, resulting in drastic psychiatric conditions.

- **Traumatic Life Events:** A person's mental health can be greatly affected by the psychological trauma, such as violence, neglect, abuse, or stress, which can increase long-lasting changes in brain functioning, such as domestic violence, sexual assault or harassment, consistent bullying in school or outside, etc. These traumas trigger the risk of PDs, greatly leading to personality disorders or post-traumatic stress disorder (PTSD).

- **Environmental Stressors:** The stress from poverty, social violence, unemployment, social isolation, academic pressure, work overload, or relationship conflicts increases the risk of the development of PD while overwhelming the mental defenses. For instance, a weak student struggling with academic pressure without support or aid may suffer from depression and anxiety, causing mental illness.

- **Hormonal Fluctuations:** Hormonal changes, especially at the time of thyroid problems, puberty, pregnancy, or maternity, can cause a change in mood and mental health due to extreme mental stress. For instance, hormonal shifts caused by childbirth result in postpartum depression.

- **Substance Abuse:** Brain chemistry is greatly damaged by drugs, certain medications, and constant consumption of alcohol, making a negative impact on the mental health of the person. These types of medications often trigger or worsen the psychiatric symptoms. For instance, psychosis can be caused in vulnerable persons by the consumption of cannabis, and mood disorders can happen due to continuous use of alcohol.

- **Social Isolation:** Loneliness and lack of emotional aid and social connection are significant predictors of anxiety, stress, and depression. Risk of depression and anxiety can happen due to emotional isolation and loneliness after a broken or unfulfilled relationship. Also, these types of PDs can be found in elderly people who live alone without social gatherings.

- **Negative Thinking Patterns & Cognitive Distortions:** Continuous negative thoughts and low coping skills can cause a greater risk of having disorders like stress, anxiety, and depression. Mental problems may occur when a person is experiencing persistent failures in life or has high expectations of oneself, creating a storm of negative thoughts and emotions.
- **Violence:** Often, soldiers or refugees who have experienced war and its difficulties, and the refugee system, suffer from different types of mental diseases, which make their lives a lot more challenging and distort their relationships.

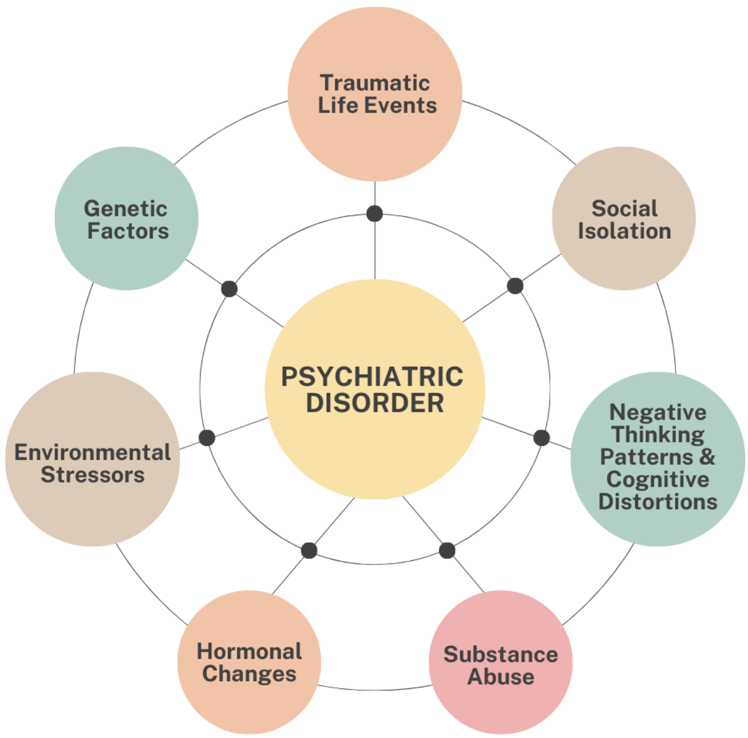

Fig. 5. Graphical Representation of Causes of Psychiatric Disorders.

The ultimate goal in this research article is to enhance the evaluation of psychiatric disorder symptoms and severity while reducing the inconsistency and uncertainty present in the causes of mental diseases, using a novel framework that is intuitionistic pentagonal fuzzy numbers.

4. A mathematical formulation of psychiatric disorder evaluation using intuitionistic pentagonal fuzzy frameworks

In this section, we are going to formulate a mathematical model for the evaluation of psychiatric disorders where the constraints of causes of these disorders are represented as IPnFNs. For the elaboration of the proposed model, four different constraints have been defined in the Table 1, which will be used for the sake of measurements, and their figurative presentation is given in Figure 6.
Here every constraint is represented in the form of intuitionistic pentagonal fuzzy number, hence we can denote the variable "Hormonal Changes" as "H" by IPnFN

Table 1. Constraints we are going to use in the model and their dimensions

Constraint	Hormonal Changes (H)	Traumatic Life Events (T)	Cognitive Distortions (C)	Environmental Stressors (E)
Dimensions	Puberty	Serious Illness	Perfectionism	Financial Instability
	Chronic Stress	Sexual Harassment	Low Self-Esteem	Unemployment
	Menstrual Cycle	Bullying Victimization	Past Failures	Academic Pressure
	Thyroid Disorders	Childhood Abuse	Negative Media Influence	Social Rejection
	Pregnancy Period	Loss of a Loved one	Overprotective Parenting	Family Conflict

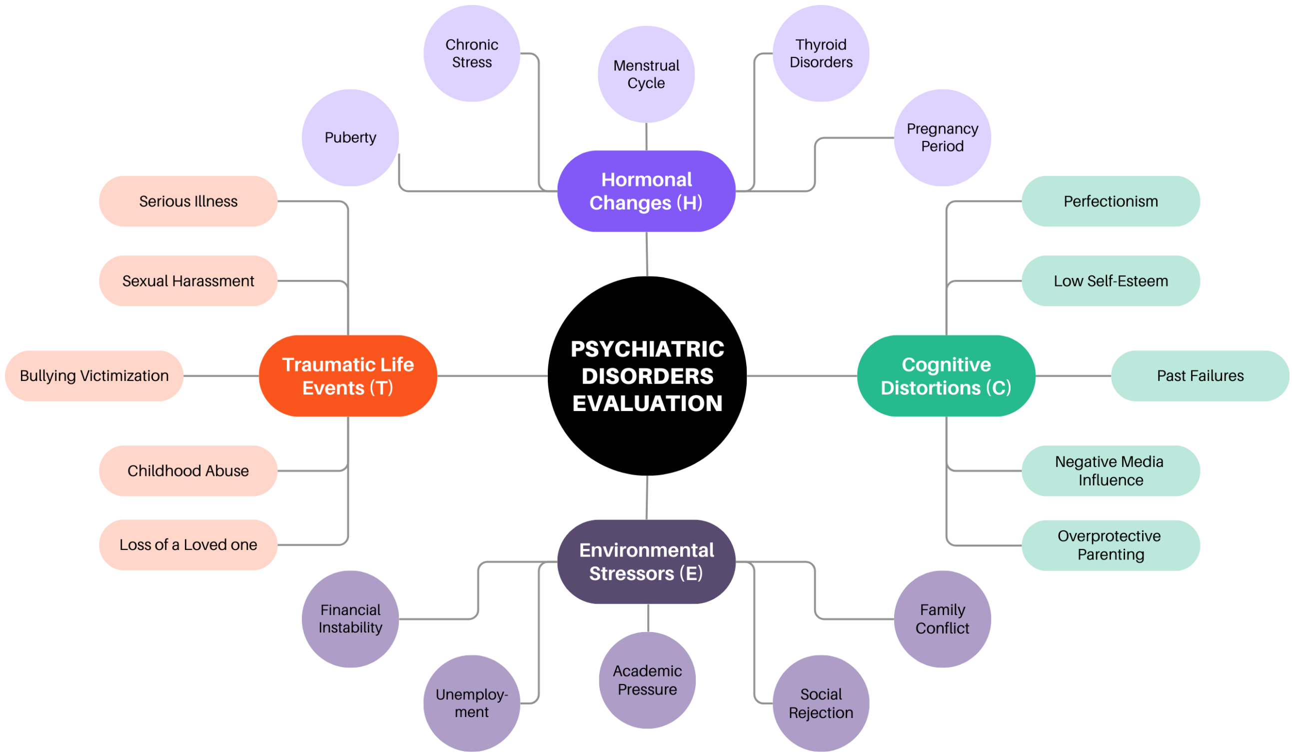

Fig. 6. Pictorial presentation of constraints we will use in the model and their dimensions.

$\mathcal{H} = \{(\alpha_H^M, \beta_H^M, \gamma_H^M, \delta_H^M, \sigma_H^M), (\alpha_H^N, \beta_H^N, \gamma_H^N, \delta_H^N, \sigma_H^N)\}$, we can represent the variable "Traumatic Life Events" as "T" by IPnFN $\mathcal{T} = \{(\alpha_T^M, \beta_T^M, \gamma_T^M, \delta_T^M, \sigma_T^M), (\alpha_T^N, \beta_T^N, \gamma_T^N, \delta_T^N, \sigma_T^N)\}$, we can indicate the variable "Cognitive Distortions" as "C" by IPnFN $\mathcal{C} = \{(\alpha_C^M, \beta_C^M, \gamma_C^M, \delta_C^M, \sigma_C^M), (\alpha_C^N, \beta_C^N, \gamma_C^N, \delta_C^N, \sigma_C^N)\}$ and we can show the variable "Environmental Stressors" as "E" by IPnFN
$\mathcal{E} = \{(\alpha_E^M, \beta_E^M, \gamma_E^M, \delta_E^M, \sigma_E^M), (\alpha_E^N, \beta_E^N, \gamma_E^N, \delta_E^N, \sigma_E^N)\}$.
Let we are taking these measures for n persons then we are going to represent them with "P_i" where $i = 1, 2, 3, ..., n$. Then we will indicate these constraints of "Hormonal Changes" as
$\mathcal{H}_i = \{(\alpha_{H_i}^M, \beta_{H_i}^M, \gamma_{H_i}^M, \delta_{H_i}^M, \sigma_{H_i}^M), (\alpha_{H_i}^N, \beta_{H_i}^N, \gamma_{H_i}^N, \delta_{H_i}^N, \sigma_{H_i}^N)\}$, we will represent the variable "Traumatic Life Events" as $\mathcal{T}_i = \{(\alpha_{T_i}^M, \beta_{T_i}^M, \gamma_{T_i}^M, \delta_{T_i}^M, \sigma_{T_i}^M), (\alpha_{T_i}^N, \beta_{T_i}^N, \gamma_{T_i}^N, \delta_{T_i}^N, \sigma_{T_i}^N)\}$, we will denote the variable "Cognitive Distortions" as $\mathcal{C}_i = \{(\alpha_{C_i}^M, \beta_{C_i}^M, \gamma_{C_i}^M, \delta_{C_i}^M, \sigma_{C_i}^M), (\alpha_{C_i}^N, \beta_{C_i}^N, \gamma_{C_i}^N, \delta_{C_i}^N, \sigma_{C_i}^N)\}$ and we will show the variable "Environmental Stressors" as $\mathcal{E}_i = \{(\alpha_{E_i}^M, \beta_{E_i}^M, \gamma_{E_i}^M, \delta_{E_i}^M, \sigma_{E_i}^M), (\alpha_{E_i}^N, \beta_{E_i}^N, \gamma_{E_i}^N, \delta_{E_i}^N, \sigma_{E_i}^N)\}$.
Each of these constraints H, T, C and E are thoroughly aggregated by the expert psychologists, suppose we represent these psychologists by e_j where $j = 1, 2, 3, ..., m$. At a given time these expert psychologists deliver evaluated data gathered from different people suppose we symbolize the data as
$\mathcal{H}_{ij} = \left\{(\alpha_{H_{ij}}^M, \beta_{H_{ij}}^M, \gamma_{H_{ij}}^M, \delta_{H_{ij}}^M, \sigma_{H_{ij}}^M), (\alpha_{H_{ij}}^N, \beta_{H_{ij}}^N, \gamma_{H_{ij}}^N, \delta_{H_{ij}}^N, \sigma_{H_{ij}}^N)\right\}$,
$\mathcal{T}_{ij} = \left\{(\alpha_{T_{ij}}^M, \beta_{T_{ij}}^M, \gamma_{T_{ij}}^M, \delta_{T_{ij}}^M, \sigma_{T_{ij}}^M), (\alpha_{T_{ij}}^N, \beta_{T_{ij}}^N, \gamma_{T_{ij}}^N, \delta_{T_{ij}}^N, \sigma_{T_{ij}}^N)\right\}$,

$\mathcal{C}_{ij} = \left\{ (\alpha^M_{C_{ij}}, \beta^M_{C_{ij}}, \gamma^M_{C_{ij}}, \delta^M_{C_{ij}}, \sigma^M_{C_{ij}}), (\alpha^N_{C_{ij}}, \beta^N_{C_{ij}}, \gamma^N_{C_{ij}}, \delta^N_{C_{ij}}, \sigma^N_{C_{ij}}) \right\}$ and
$\mathcal{E}_{ij} = \left\{ (\alpha^M_{E_{ij}}, \beta^M_{E_{ij}}, \gamma^M_{E_{ij}}, \delta^M_{E_{ij}}, \sigma^M_{E_{ij}}), (\alpha^N_{E_{ij}}, \beta^N_{E_{ij}}, \gamma^N_{E_{ij}}, \delta^N_{E_{ij}}, \sigma^N_{E_{ij}}) \right\}$. We can also categorize the level of expertise of the expert psychologists with the associated weights denoted by $\mathfrak{W}_j$ with $j = 1, 2, 3..., m$, in the Table 2 as follows,
A knowledge and experience based evaluation of the psychologists has been made to categorize them

Table 2. The grades of expertise level and their numerical interpretation as weights of each psychologist

Expertise level	Weights ($\mathfrak{W}_j$)
Low	0.25
Moderate	0.5
High	0.75
Very High	0.95

and assign them weights according to their level, considering the constraints we are dealing with in the required amount of time. We are going to approach and consult the expert psychologists with higher expertise in the constraints we are dealing with, to have more robust and accurate results of the aggregation of the proposed model. So the weighted arithmetic means of the constraints gave us the following mathematical formulations,

$$\mathcal{H}_i = \left\{ \begin{array}{c} \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^M_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^M_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^M_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^M_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^M_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right), \\ \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^N_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^N_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^N_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^N_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^N_{H_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right) \end{array} \right\}, \tag{22}$$

$$\mathcal{T}_i = \left\{ \begin{array}{c} \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^M_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^M_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^M_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^M_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^M_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right), \\ \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^N_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^N_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^N_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^N_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^N_{T_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right) \end{array} \right\}, \tag{23}$$

$$\mathcal{C}_i = \left\{ \begin{array}{c} \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^M_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^M_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^M_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^M_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^M_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right), \\ \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^N_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^N_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^N_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^N_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^N_{C_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right) \end{array} \right\}, \tag{24}$$

$$\mathcal{E}_i = \left\{ \begin{array}{c} \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^M_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^M_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^M_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^M_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^M_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right), \\ \left(\frac{\sum_{j=1}^{m} \mathfrak{W}_j \alpha^N_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \beta^N_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \gamma^N_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \delta^N_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j}, \frac{\sum_{j=1}^{m} \mathfrak{W}_j \sigma^N_{E_{ij}}}{\sum_{j=1}^{m} \mathfrak{W}_j} \right) \end{array} \right\}. \tag{25}$$

After obtaining these results we are going to formulate a table of the evaluated values which will allow us to aggregate further evaluations of the proposed model more conveniently. For the sake of aggregation two logical equations have been formulated that are given as,

$$\mathcal{L} = \bigcap_{\forall i \in P} \left\{ (\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{E}_i) \wedge_I (\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{T}_i) \right\}, \tag{26}$$

$$\mathcal{K} = \bigcup_{\forall i \in P} \left\{ (\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{E}_i) \wedge_I (\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{T}_i) \right\}. \tag{27}$$

We can explain the Equations 26 and 27, such that two logical predicates are utilized in the evaluation process. The first statement is currently correct when the membership of all factors exceeds a certain threshold, non-membership is low, and the average magnitude of fuzzy numbers exceeds a certain threshold. In this case, the patient is considered to be suffering from a mental disorder. The second case is currently active, when there are severe symptoms in any of the elements, this takes into account not only the severity of the diagnosis, but also the interplay of different symptoms. These equations will show the psychologists to focus on the causes that are the most significant and affect the human mind. Furthermore, $\wedge_I$ and $\rightarrow_I$ are referred to as the intuitionistic fuzzy logical conjunction and implication, respectively. Additionally, $\bigcap_{\forall i \in P} x_i = \wedge_I x_i$ and $\bigcup_{\forall i \in P} x_i = \vee_I x_i$, here each x_i is an intuitionistic pentagonal fuzzy number. Note that the intuitionistic pentagonal disjunction,

$$\left\{ (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M), (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N) \right\} \vee_I \left\{ (\alpha_2^M, \beta_2^M, \gamma_2^M, \delta_2^M, \sigma_2^M), (\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N) \right\}$$

$$= \left\{ \begin{array}{c} max(\alpha_1^M, \alpha_2^M), max(\beta_1^M, \beta_2^M), max(\gamma_1^M, \gamma_2^M), max(\delta_1^M, \delta_2^M), max(\sigma_1^M, \sigma_2^M), \\ min(\alpha_1^N, \alpha_2^N), min(\beta_1^N, \beta_2^N), min(\gamma_1^N, \gamma_2^N), min(\delta_1^N, \delta_2^N), min(\sigma_1^N, \sigma_2^N) \end{array} \right\}, \tag{28}$$

the intuitionistic pentagonal conjunction

$$\left\{ (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M), (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N) \right\} \wedge_I \left\{ (\alpha_2^M, \beta_2^M, \gamma_2^M, \delta_2^M, \sigma_2^M), (\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N) \right\}$$

$$= \left\{ \begin{array}{c} min(\alpha_1^M, \alpha_2^M), min(\beta_1^M, \beta_2^M), min(\gamma_1^M, \gamma_2^M), min(\delta_1^M, \delta_2^M), min(\sigma_1^M, \sigma_2^M), \\ max(\alpha_1^N, \alpha_2^N), max(\beta_1^N, \beta_2^N), max(\gamma_1^N, \gamma_2^N), max(\delta_1^N, \delta_2^N), max(\sigma_1^N, \sigma_2^N) \end{array} \right\}, \tag{29}$$

the intuitionistic pentagonal negation

$$\neg_N \begin{Bmatrix} (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M), \\ (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N) \end{Bmatrix} = \begin{Bmatrix} (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N), \\ (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M) \end{Bmatrix}, \tag{30}$$

and the intuitionistic pentagonal implication

$$\begin{Bmatrix} (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M), \\ (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N) \end{Bmatrix} \rightarrow_I \begin{Bmatrix} (\alpha_2^M, \beta_2^M, \gamma_2^M, \delta_2^M, \sigma_2^M), \\ (\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N) \end{Bmatrix}$$

$$= \neg_N \begin{Bmatrix} (\alpha_1^M, \beta_1^M, \gamma_1^M, \delta_1^M, \sigma_1^M), \\ (\alpha_1^N, \beta_1^N, \gamma_1^N, \delta_1^N, \sigma_1^N) \end{Bmatrix} \vee_I \begin{Bmatrix} (\alpha_2^M, \beta_2^M, \gamma_2^M, \delta_2^M, \sigma_2^M), \\ (\alpha_2^N, \beta_2^N, \gamma_2^N, \delta_2^N, \sigma_2^N) \end{Bmatrix}. \tag{31}$$

5. Numerical examples

Here, we are going to formulate a numerical example by aggregating our results on a practical implication of psychological disorder to show the applicability of the proposed technique and elaborate on its practical scope in real-world scenarios. Suppose we have to determine the risk of various causes ($\mathcal{H}_i$, $\mathcal{T}_i$, $\mathcal{C}_i$ and $\mathcal{E}_i$), given in Table 1 for which three expert psychologists $\mathsf{e} = \{\mathsf{e}_1, \mathsf{e}_2, \mathsf{e}_3\}$, have given the information numerically by examining two patients (serial number 3 and 5) and using Equations 22, 23, 24 and 25, the aggregated values are stated in Table 3. Here $P_3 = (\mathcal{H}_3, \mathcal{T}_3, \mathcal{C}_3, \mathcal{E}_3)$ and $P_5 = (\mathcal{H}_5, \mathcal{T}_5, \mathcal{C}_5, \mathcal{E}_5)$, denote the patient number 3 and 5. For convenience's sake, we have taken only two patients; the proposed method is applicable to several persons. Also, we will evaluate the data for both patients and determine the results on the basis of the given information. In addition, the expert psychologists e_1, e_2, and e_3 are taken as having high, very high, and very high levels of expertise, respectively. Therefore, we get the weights as $\mathfrak{W}_1 = 0.75$ and $\mathfrak{W}_2 = \mathfrak{W}_3 = 0.95$, for each of the expert psychologist.

Now, by utilizing Equations 26 and 27 the aggregated conjunctions and disjunctions between constraints are given in Table 4.

Table 3. Initial data gathered from expert psychologists and evaluated values

Psychologist/ *Constraint*	e_1	e_2	e_3	Evaluated Values
$\mathcal{H}_3$	$\begin{pmatrix}(0.12, 0.16, 0.55, 0.72, 0.82), \\ (0.02, 0.32, 0.39, 0.41, 0.45)\end{pmatrix}$	$\begin{pmatrix}(0.22, 0.28, 0.48, 0.65, 0.73), \\ (0.21, 0.32, 0.32, 0.56, 0.97)\end{pmatrix}$	$\begin{pmatrix}(0.54, 0.56, 0.63, 0.77, 0.81), \\ (0.28, 0.37, 0.41, 0.52, 0.53)\end{pmatrix}$	$\begin{pmatrix}(0.31, 0.35, 0.55, 0.71, 0.78), \\ (0.18, 0.34, 0.37, 0.50, 0.66)\end{pmatrix}$
$\mathcal{H}_5$	$\begin{pmatrix}(0.02, 0.41, 0.47, 0.51, 0.72), \\ (0.15, 0.26, 0.45, 0.47, 0.83)\end{pmatrix}$	$\begin{pmatrix}(0.22, 0.38, 0.50, 0.55, 0.84), \\ (0.38, 0.44, 0.59, 0.69, 0.76)\end{pmatrix}$	$\begin{pmatrix}(0.24, 0.39, 0.55, 0.68, 0.89), \\ (0.28, 0.55, 0.67, 0.72, 0.81)\end{pmatrix}$	$\begin{pmatrix}(0.17, 0.39, 0.51, 0.58, 0.82), \\ (0.28, 0.43, 0.58, 0.64, 0.80)\end{pmatrix}$
$\mathcal{T}_3$	$\begin{pmatrix}(0.45, 0.55, 0.75, 0.77, 0.85), \\ (0.23, 0.33, 0.53, 0.63, 0.82)\end{pmatrix}$	$\begin{pmatrix}(0.14, 0.27, 0.31, 0.47, 0.58), \\ (0.23, 0.39, 0.45, 0.47, 0.82)\end{pmatrix}$	$\begin{pmatrix}(0.17, 0.21, 0.23, 0.75, 0.79), \\ (0.44, 0.56, 0.62, 0.79, 0.82)\end{pmatrix}$	$\begin{pmatrix}(0.24, 0.33, 0.41, 0.66, 0.73), \\ (0.30, 0.43, 0.53, 0.63, 0.82)\end{pmatrix}$
$\mathcal{T}_5$	$\begin{pmatrix}(0.02, 0.41, 0.44, 0.46, 0.80), \\ (0.36, 0.49, 0.57, 0.81, 0.83)\end{pmatrix}$	$\begin{pmatrix}(0.11, 0.21, 0.46, 0.79, 0.84), \\ (0.27, 0.33, 0.48, 0.64, 0.71)\end{pmatrix}$	$\begin{pmatrix}(0.11, 0.14, 0.16, 0.54, 0.66), \\ (0.48, 0.65, 0.72, 0.93, 0.95)\end{pmatrix}$	$\begin{pmatrix}(0.08, 0.24, 0.35, 0.61, 0.76), \\ (0.37, 0.49, 0.59, 0.79, 0.83)\end{pmatrix}$
$\mathcal{C}_3$	$\begin{pmatrix}(0.19, 0.24, 0.56, 0.87, 0.89), \\ (0.14, 0.27, 0.45, 0.64, 0.88)\end{pmatrix}$	$\begin{pmatrix}(0.22, 0.28, 0.49, 0.51, 0.75), \\ (0.38, 0.50, 0.53, 0.67, 0.76)\end{pmatrix}$	$\begin{pmatrix}(0.14, 0.21, 0.27, 0.35, 0.51), \\ (0.29, 0.48, 0.66, 0.69, 0.71)\end{pmatrix}$	$\begin{pmatrix}(0.18, 0.24, 0.43, 0.55, 0.70), \\ (0.28, 0.43, 0.55, 0.67, 0.78)\end{pmatrix}$
$\mathcal{C}_5$	$\begin{pmatrix}(0.19, 0.26, 0.41, 0.65, 0.88), \\ (0.16, 0.31, 0.58, 0.75, 0.92)\end{pmatrix}$	$\begin{pmatrix}(0.23, 0.24, 0.32, 0.39, 0.67), \\ (0.41, 0.49, 0.53, 0.76, 0.79)\end{pmatrix}$	$\begin{pmatrix}(0.19, 0.35, 0.62, 0.77, 0.85), \\ (0.22, 0.29, 0.46, 0.58, 0.71)\end{pmatrix}$	$\begin{pmatrix}(0.20, 0.28, 0.45, 0.60, 0.79), \\ (0.27, 0.37, 0.52, 0.69, 0.80)\end{pmatrix}$
$\mathcal{E}_3$	$\begin{pmatrix}(0.29, 0.35, 0.43, 0.44, 0.59), \\ (0.36, 0.53, 0.59, 0.62, 0.64)\end{pmatrix}$	$\begin{pmatrix}(0.17, 0.25, 0.27, 0.41, 0.72), \\ (0.22, 0.45, 0.50, 0.58, 0.70)\end{pmatrix}$	$\begin{pmatrix}(0.26, 0.34, 0.44, 0.59, 0.69), \\ (0.17, 0.21, 0.49, 0.51, 0.56)\end{pmatrix}$	$\begin{pmatrix}(0.24, 0.31, 0.38, 0.48, 0.67), \\ (0.24, 0.39, 0.52, 0.57, 0.63)\end{pmatrix}$
$\mathcal{E}_5$	$\begin{pmatrix}(0.02, 0.38, 0.41, 0.50, 0.55), \\ (0.11, 0.22, 0.33, 0.55, 0.77)\end{pmatrix}$	$\begin{pmatrix}(0.33, 0.43, 0.53, 0.54, 0.65), \\ (0.23, 0.35, 0.41, 0.71, 0.84)\end{pmatrix}$	$\begin{pmatrix}(0.34, 0.36, 0.48, 0.66, 0.94), \\ (0.17, 0.22, 0.37, 0.58, 0.66)\end{pmatrix}$	$\begin{pmatrix}(0.25, 0.39, 0.48, 0.57, 0.73), \\ (0.17, 0.27, 0.37, 0.62, 0.76)\end{pmatrix}$

Hence, the final results from Equations 26 and 27 are given as,

$$\mathcal{L} = \begin{Bmatrix} (0.28, 0.43, 0.55, 0.67, 0.78), \\ (0.18, 0.28, 0.45, 0.58, 0.79) \end{Bmatrix}, \mathcal{K} = \begin{Bmatrix} (0.28, 0.43, 0.58, 0.69, 0.80), \\ (0.17, 0.24, 0.43, 0.55, 0.70) \end{Bmatrix}$$

Table 4. Evaluation of logical predicates

Operators/Patient number	P_3	P_5
$\mathcal{H}_i \wedge_I \mathcal{C}_i$	$\begin{pmatrix}(0.18, 0.24, 0.43, 0.55, 0.70),\\(0.28, 0.43, 0.55, 0.67, 0.78)\end{pmatrix}$	$\begin{pmatrix}(0.17, 0.28, 0.45, 0.58, 0.79),\\(0.28, 0.43, 0.58, 0.69, 0.80)\end{pmatrix}$
$\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{E}_i$	$\begin{pmatrix}(0.28, 0.43, 0.55, 0.67, 0.78),\\(0.18, 0.24, 0.43, 0.55, 0.63)\end{pmatrix}$	$\begin{pmatrix}(0.28, 0.43, 0.58, 0.69, 0.80),\\(0.17, 0.27, 0.37, 0.58, 0.76)\end{pmatrix}$
$\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{T}_i$	$\begin{pmatrix}(0.28, 0.43, 0.55, 0.67, 0.78),\\(0.18, 0.24, 0.43, 0.55, 0.70)\end{pmatrix}$	$\begin{pmatrix}(0.28, 0.43, 0.58, 0.69, 0.80),\\(0.17, 0.28, 0.45, 0.58, 0.79)\end{pmatrix}$
$(\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{E}_i) \wedge_I (\mathcal{H}_i \wedge_I \mathcal{C}_i \rightarrow_I \mathcal{T}_i)$	$\begin{pmatrix}(0.28, 0.43, 0.55, 0.67, 0.78),\\(0.18, 0.24, 0.43, 0.55, 0.70)\end{pmatrix}$	$\begin{pmatrix}(0.28, 0.43, 0.58, 0.69, 0.80),\\(0.17, 0.28, 0.45, 0.58, 0.79)\end{pmatrix}$

We can convert both of these IPnFNs into real numerical numbers by utilizing the score function defined in the Definition 10, giving us the solutions $\mathcal{L} = 0.086$ and $\mathcal{K} = 0.138$. Since score function values of IPnFNs lie in the interval [-1,1], we can assume that the value of $\mathcal{L}$ is approximately moderate and the value of $\mathcal{K}$ is a little bit higher than moderate. Therefore, these results yield us the conclusion that more than average people are getting affected by the psychological disorders due to specific causes, whether these are just for a short amount of time or permanent.

6. Conclusion

Psychiatric disorders are a complex and profound aspect of human health, whose full understanding and accurate diagnosis have always been a challenge. The uncertain nature of psychiatric disorder symptoms, fluctuating severity, and the individual psychological state of each patient make the process more difficult. Against such a backdrop, the use of intuitionistic pentagonal fuzzy numbers (IPnFNs) in psychological assessment has emerged as a modern and revolutionary development. In this research article, we looked in detail at how the mathematical model of IPnFNs makes the diagnosis of psychiatric disorders more objective, transparent, and accurate. Especially when we take into account the multifactorial causes associated with various psychological problems, such as depression, anxiety, etc., the aspects of severity, reluctance, acceptance, and non-acceptance of these causes can be effectively modeled through IPnFNs. The proposed model helps us to understand that a patient's condition can be influenced not only by outward symptoms but also by hidden mental disorders. During the research process, it became clear that the traditional diagnostic methods have flaws that can be greatly reduced in the mathematical structure of IPnFNs. This five-point framework goes deep into mental illness to create a picture that is more complete and realistic. Furthermore, the use of IPnFNs has not only been limited to improving existing diagnoses but has also opened up new avenues for mental health treatment and care. When psychiatrists get a numerical and reliable analysis of different aspects of a patient, they can prescribe more accurate, personalized, and effective treatments. IPnFNs have thus shifted mental health care from the old principle of "one size fits all" to a more individualized and patient-centered approach. Another important aspect that came out in this study was that factors such as uncertainty, hesitation, and ambiguous causes can be easily modeled through IPnFNs. In psychiatric disorders, the patient's condition is often not completely clear, and some symptoms are faint or temporary. In such a situation, the physical logic of IPnFNs is of great help in sorting out this fog and drawing more reliable conclusions. During the research study, we also realized that the IPnFNs model can be applied not only to the patient's condition, but also to the patient's environment, past experiences, social pressures, family background, and even current economic or social conditions. In the final analysis, it is fair to say that the use of intuitionistic pentagonal fuzzy numbers is opening a new horizon in the world of mental health. It is a balanced bridge between mathematical rigor and the fragility of the human psyche, while the model provides us with a numerically objective analysis; on the other hand, it also respectfully encapsulates the complex web of human experiences, emotions, and personal stories. This research is not only evidence of the effectiveness of IPnFNs, but it also highlights the fact that mental health care in the future will be more scientific, personalized, and people-centric. If we adopt these mathematical models wisely and combine them with human compassion, social consciousness, and technology, we can achieve a major and lasting success against psychological disorders.

6.1 Future directions.

The study also identified several bright prospects for the future.

- By analyzing large-scale patient data, new patterns of symptoms can be identified, and psychological disorders can be predicted in the early stages.
- When advanced technologies such as artificial intelligence (AI) and machine learning are integrated with IPnFNs, psychological assessment can be more automated, accurate, and faster.
- In the future, mental health management can be made more accessible and effective for the general public by integrating IPnFNs into digital platforms, mobile applications, and real-time monitoring systems for mental health.
- In developing countries where there is a shortage of psychiatrists, this technology can revolutionize the mental health system.

6.2 Limitations

However, the study also highlighted that the use of IPnFNs is not without some limitations.

- The success of the proposed model depends on the accuracy of the data provided.
- If the patient does not describe his condition with complete truth, or the psychiatrist does not understand the causes correctly, the mathematical model can also give misleading results.
- Furthermore, to use IPnFNs, psychologists require specific mathematical training and logical analysis skills, which are not available to every professional clinician.

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Conflicts of Interest

The authors declare no conflicts of interest.

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