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Fuzzy Compromise Approach for Solving Stochastic Multi-objective q-Rung Orthopair Fuzzy Linear Programming Problem

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ABSTRACT

The model considered in this study is a multi-objective linear programming (MOLP) model under uncertainty, in which the coefficients of the objective functions are represented by q-rung orthopair fuzzy numbers (q-ROFNs), while the right-hand side constraint parameters b_i are treated as probabilistic quantities. The random variables b_i are assumed to follow known probability distributions and are characterized by specified means and variances. By employing an appropriate score function and considering several probability distributions, namely Gamma, log-normal, and exponential distributions, the initial probabilistic q-rung orthopair fuzzy (q-ROF) MOLP problem is transformed into an equivalent deterministic MOLP model. The Zimmermann methodology with linear membership degrees (MDs) is then applied to represent the preferences of the decision-maker and obtain a satisfactory compromise solution. A numerical example is provided to demonstrate the applicability and efficiency of the proposed methodology. The study concludes with final observations and suggestions for future research.

1. Introduction

A MOLP Problem represents an important class of models within mathematical optimization. Unlike a conventional linear programming problem (LPP), which focuses on a single objective, a MOLP problem involves multiple objective functions that are optimized concurrently. These objectives, often conflicting in nature, may require simultaneous maximization or minimization while satisfying a shared system of constraints. Huang *et al.,* [1] proposed an interactive fuzzy framework for multi-objective optimization (MOO) grounded in the concept of pareto optimality. Tayebikhorami *et al.,* [2] developed an approach for wastewater treatment based on a MOO framework. Their model enabled the identification of suitable alternatives by analyzing solutions along the Pareto frontier, ensuring that multiple performance criteria were simultaneously satisfied. Li [3] introduced a solution procedure for MOLP models where uncertainty is captured extended fuzzy versions.

In this formulation, the parameters associated with both the objective functions and the constraint system are expressed using this fuzzy representation. Rouhbakhsh *et al.,* [4] presented a

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multi-objective programming (MOP) framework within a hesitant fuzzy setting, where the assessment data supplied by decision makers is modeled using hesitant fuzzy information to capture uncertainty and hesitation in evaluations. Ahmed [5] developed a solution strategy for multi-objective problems characterized by intuitionistic fuzzy parameters. In recent years, MOP under uncertain environments has attracted considerable attention, with numerous contributions addressing different modeling and solution perspectives (see, for instance, Groetzner and Werner [6]; Ricciardi *et al.,* [7], Karimi *et al.,* [8], Vllamil *et al.,* [9], Huang *et al.,* [10], Liu [11], Xia *et al.,* [12], Chakong and Haimes [13] ,Hernandez- Jimenez [14], Ishibuchi, and Tanaka [15], and Lachhwani [16]). Jana *et al.,* [17] proposed a family of q-ROF Dombi aggregation operators derived from Dombi operational laws. Their work introduced several operators; these aggregation mechanisms were further utilized to construct decision-making (DM) models for multiple-attribute problems in uncertain settings. Liu and Wang [18] presented a detailed analysis of operational rules for q-ROF sets using the ATT-based approach. Subsequently, Liu, P., and Liu, W. [19] extended the theory by defining linguistic q-ROFNs, where MDs and non-membership degree (NMD) are represented through linguistic variables. Liu *et al.,* [20] later developed an improved methodology for group DM based on q-ROF information, emphasizing reduced information distortion and enhanced evaluation reliability without relying heavily on conventional aggregation procedures. Zhong *et al.,* [21] introduced Dombi power partitioned Heronian mean operators for q-ROFNs, which served as the foundation for a DM framework. Farid and Riaz [22] underscored the role of ergonomics in establishing effective human–environment interactions by incorporating anatomical and psychological considerations. They proposed a decision support system for glove design integrating quality function deployment and q-ROF modeling to address uncertainty in customer preferences and expert assessments. More recently, research on q-ROF theory continues to expand, as reflected in contemporary studies (e.g., Wang *et al.,* [23]; Guliyev *et al.,* [24]; Ullah *et al.,* [25]).

Although significant progress has been made in MOLP under uncertainty and q-ROF environments, existing studies often focus on either probabilistic or fuzzy parameters separately, and few approaches integrate uncertain objective function coefficients with probabilistic constraint parameters while providing a practical compromise solution. Moreover, many aggregation operators for q-ROF sets lack flexibility in reflecting decision-maker preferences. To address these gaps, this study develops a probabilistic q-ROF-MOLP model with fuzzy objective coefficients and stochastic constraints, transforms it into a deterministic framework using a score function and various probability distributions, and applies Zimmermann's approach with linear MDs to derive an optimal compromise solution. A numerical example demonstrates the effectiveness and applicability of the proposed methodology, offering a systematic tool for DM in uncertain and fuzzy multi-objective environments.

This study is structured into seven distinct parts. The second section reviews the fundamental concepts and definitions required for the subsequent analysis. The problem formulation is detailed in Section 3, where the MOLP model is developed by representing the objective coefficients through q-ROFNs, while the constraint parameters on the right-hand side are considered as stochastic variables characterized by known statistical properties. Section 4 describes the solution framework adopted in the study. To demonstrate the practicality of the proposed approach, Section 5 presents a numerical illustration. A comparative evaluation with relevant existing techniques is carried out in Section 6. The final section summarizes the main findings and highlights potential avenues for further investigation.

2. Preliminaries

This section outlines the key concepts and fundamental notions associated with q-ROFNs. It also describes the basic operational rules and introduces the ranking mechanism employed for their evaluation and comparison.

Definition 1 [26]. Consider H be a finite set and let ζ be an arbitrary component of H. A fuzzy set (FS) $\tilde{A}$ on H is characterized by the collection

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in H\},$$

here $\mu_{\tilde{A}}(\zeta): H \to [0,1]$. This value represents the degree to which x belongs to the fuzzy set $\tilde{A}$, and $\mu_{\tilde{A}}$ is known as the MD.

Definition 2 [26]. Let Rrepresent the set of real numbers. A fuzzy set $\tilde{A}$defined on R, characterized by a membership function (MF) $\mu_{\tilde{A}}: R \to [0,1]$, is termed as a fuzzy number (FN) provided that the following properties are satisfied:

Normality: There exists at least one point $x^0 \in R$for which $\mu_{\tilde{A}}(x^0) = 1$.

Convexity: For any $x, y \in R$and any $\gamma \in [0,1]$, the MF obeys $\mu_{\tilde{A}}(\gamma x + (1-\gamma)y) \geq \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\right)$.

Piecewise Continuity: The function $\mu_{\tilde{A}}(x)$is piecewise continuous across the domain R.

Definition 3 [27]. Consider a universe of discourse H. A q-ROF set, symbolized by Δ, is defined as $\Delta = \{\langle x, \mu_\Delta(x), \nu_\Delta(x)\rangle \mid x \in H\}$, where the mappings $\mu_\Delta: H \to [0,1]$and $\nu_\Delta: H \to [0,1]$denote the MD and non-MDs (NMD) of the element x, respectively. These degrees must satisfy the restriction $(\mu_\Delta(x))^q + (\nu_\Delta(x))^q \leq 1, q \geq 1$. The hesitancy degree corresponding to $x$is expressed as$\chi_\Delta = (1 - (\mu_\Delta)^q - (\nu_\Delta)^q)^{1/q}$.

The membership space q-ROF set for different values of q is presented in Figure 1.

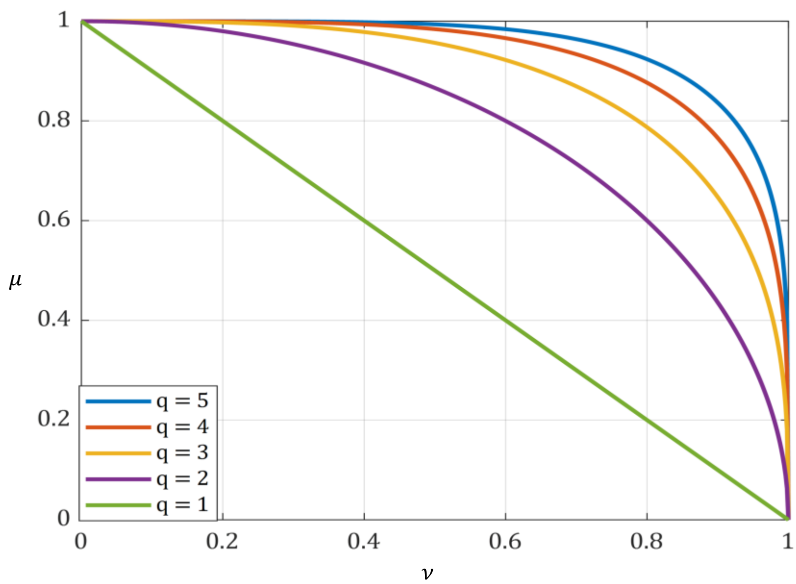

Fig. 1. Diagrammatic representation of q- ROFN

Definition 4. [20]. A triangular q-ROFN (TQROPFN) can be expressed as

$$\tilde{\Theta}^{QRF} = \langle (a_1, a_2, a_3); (\mu_{\tilde{\Theta}}, \nu_{\tilde{\Theta}})^q \rangle,$$

where the real-valued parameters satisfy $a_1 \leq a_2 \leq a_3$. The membership and non-membership degrees are described through piecewise functions defined over the real line:

$$\mu_{\tilde{\Theta}}(x)=\begin{cases}\mu_a\left(\dfrac{x-a_1}{a_2-a_1}\right), & x\in[a_1,a_2),\\ \mu_a, & x=a_2,\\ \mu_a\left(\dfrac{a_3-x}{a_3-a_2}\right), & x\in[a_2,a_3),\\ 0, & \text{otherwise},\end{cases}$$

$$\nu_{\tilde{\Theta}}(x)=\begin{cases}\nu_a\left(\dfrac{x-a_1}{a_2-a_1}\right), & x\in[a_1,a_2),\\ \nu_a, & x=a_2,\\ \nu_a\left(\dfrac{a_3-x}{a_3-a_2}\right), & x\in[a_2,a_3),\\ 0, & \text{otherwise},\end{cases}$$

where μ_a and ν_a represent the highest admissible MD and NMDs, respectively. These parameters belong to the interval $[0, 1]$and must satisfy the orthopair condition

$$\mu_a^q+\nu_a^q\leq 1.$$

The quantities $\mu_a$and $\nu_a$quantify the degrees of support and opposition inherent in the fuzzy representation.

Definition 5. [20]. Consider two triangular q-ROFNs (TQROPFNs)

$$\Delta^{QRF}=\langle(\gamma_1,\gamma_2,\gamma_3);(\mu_\Delta,\nu_\Delta)^q\rangle \text{and } \tilde{b}^{QRF}=\langle(\lambda_1,\lambda_2,\lambda_3);(\mu_\Delta,\nu_\Delta)^q\rangle.$$

The fundamental operational laws for these fuzzy numbers are defined as follows:

(i) Addition

$$\Delta^{QRF}\oplus\nabla^{QRF}=\langle(\gamma_1+\lambda_1,\gamma_2+\lambda_2,\gamma_3+\lambda_3);\left(\mu_{\tilde{a}}^q+\mu_{\tilde{b}}^q-\mu_\Delta^q\mu_\nabla^q\right)^{1/q},\nu_\Delta\nu_\nabla\rangle.$$

(ii) Multiplication

$$\Delta^{QRF}\otimes\nabla^{QRF}=\langle(\gamma_1\lambda_1,\gamma_2\lambda_2,\gamma_3\lambda_3);\mu_\Delta\mu_\nabla,\left(\nu_\Delta^q+\nu_\nabla^q-\nu_\Delta^q\nu_\nabla^q\right)^{1/q}\rangle.$$

(iii) Division

$$\Delta^{QRF}\oslash\nabla^{QRF}=\langle\left(\frac{\gamma_1}{\lambda_3},\frac{\gamma_2}{\lambda_2},\frac{\gamma_3}{\lambda_1}\right);\mu_\Delta\mu_\nabla,\left(\nu_\Delta^q+\nu_\nabla^q-\nu_\Delta^q\nu_\nabla^q\right)^{1/q}\rangle.$$

(iv) Scalar Multiplication

$$\delta\Delta^{QRF}=\langle(\delta\gamma_1,\delta\gamma_2,\delta\gamma_3);\left(1-(1-\mu_\Delta^q)^\delta\right)^{1/q},\nu_\nabla^\delta\rangle,\delta>0.$$

(v) Power Operation

$$\left(\Delta^{QRF}\right)^\delta=\langle(\gamma_1^\delta,\gamma_2^\delta,\gamma_3^\delta);\mu_\Delta^k,\left(1-(1-\nu_\Delta^q)^\delta\right)^{1/q}\rangle,\delta>0.$$

Definition 6. [20]. Let $\Delta^{QRF}=\langle(\gamma_1,\gamma_2,\gamma_3);\mu_\Delta,\nu_\Delta\rangle$ denote a triangular q-ROFN (TQROPFN). The score function, which is used to assess and compare such FNs, is defined as

$$SC(\Delta^{QRF})=\frac{(\gamma_1+\gamma_2+\gamma_3)}{8}\left(1+\mu_\Delta^q-\nu_\Delta^q\right).$$

Definition 7. [20]. Consider two TQROPFNs $\Delta^{QRF}=\langle(\gamma_1,\gamma_2,\gamma_3);\mu_\Delta,\nu_\Delta\rangle$ and $\nabla^{QRF}=\langle(\lambda_1,\lambda_2,\lambda_3);\mu_\nabla,\nu_\nabla\rangle$. The comparative relation between these q-ROFNs is established through their score values. Specifically,

i. Δ^{QRF} is regarded as superior (or inferior) to ∇^{QRF} if
$SC(\Delta^{QRF})>SC(\nabla^{QRF})$(or $SC(\Delta^{QRF})<SC(\nabla^{QRF})$).

ii. The two fuzzy numbers are considered equivalent whenever their score functions coincide, that is,

$SC(\Delta^{QRF}) = SC(\nabla^{QRF})$.

A MOLP problem can be expressed as follows:

Objective functions:

Maximize $F^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j, k = 1,2, \dots, K,$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1,2, \dots, m,$$

where x_j represents the decision variables, $c_j^{(k)}$ are the coefficients corresponding to the k-th objective function, a_{ij} are the constraint coefficients, and b_i denotes the upper bounds of the constraints.

Definition 8 [28]. Let $\bar{X} \in G$ be a feasible solution, where G denotes the feasible region of a MOLP problem. The vector $\bar{X}$ is said to be an efficient (or Pareto-optimal) solution if and only if there does not exist another feasible vector $X \in G$ such that

$$\sum_{j=1}^{n} c_j^{(k)} x_j \leq \sum_{j=1}^{n} c_j^{(k)} \bar{x}_j \text{for all } k = 1,2, \dots, r$$

with at least one strict inequality for some k. In other words, no other feasible solution can improve any objective without worsening at least one of the remaining objectives.

Definition 9. [29]. A feasible vector $\mathrm{X}^0 \in \mathrm{G}$ is said to be a compromise $\Leftrightarrow$ $\mathrm{X}^0 \in \mathrm{H}$ and $\mathrm{Z}(\mathrm{X}^0) \leq \wedge_{\mathrm{X} \in \mathrm{G}} \mathrm{Z}(\mathrm{X})$, where $\wedge$ stands for "minimum" and H is the set of efficient solutions.

4. Problem Statement and Solution Concepts

A q-ROF-MOLPP with probabilistic right-hand side (RHS) constraints can be formulated as:

(P) Maximize $\tilde{F}_{QROF}^{(k)} = \sum_{j=1}^{n} (\tilde{c}_j^{(k)})_{QROF} x_j,$

subject to $\Pr \left(\sum_{j=1}^{n} a_{ij} x_j \leq b_i \right) \geq 1 - \zeta_i, i = 1,2, \dots, m,$

$$x_j \geq 0, j = 1,2, \dots, n,$$

where $(\tilde{c}_j^{(k)})_{QROF}$ denotes a q-ROFN, a_{ij} and b_i are random variables, $\zeta_i \in (0,1)$ are specified probabilities, and the decision variables x_j are deterministic.

The problem can be converted into a deterministic equivalent using the score function of the q-ROFNs and by assuming a specific probability distribution for the RHS variables. The most considered distributions include:

i. Uniform distribution
ii. Exponential distribution
iii. Gamma distribution
iv. Log-normal distribution

4.1 Deterministic Equivalents for Different Distributions

Uniform Distribution: If $b_i$is uniformly distributed over $[\sigma_i, \varrho_i]$, its probability density function (PDF) is

$$\chi(b_i) = \begin{cases} \dfrac{1}{\varrho_i - \sigma_i}, & \sigma_i \le b_i \le \varrho_i, \\ 0, & \text{otherwise}, \end{cases}$$

with mean $\mu = (\varrho_i + \sigma_i)/2$and variance Var $= (\varrho_i - \sigma_i)^2/12$.

The probabilistic constraint becomes $\sum_{j=1}^{n} a_{ij}\, x_j \le \sigma_i + \zeta_i(\varrho_i - \sigma_i)$. Thus, the deterministic problem is (P1) Maximize $F^{(k)}(x) = SC\left(\sum_{j=1}^{n}(\tilde{c}_j^{(k)})_{QROF}\right)x_j$,

subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le \sigma_i + \zeta_i(\varrho_i - \sigma_i), x_j \ge 0.$$

Exponential Distribution: For exponential $b_i$with rate η_i, the PDF is $\chi(b_i) = \eta_i e^{-\eta_i b_i}, \mu = 1/\eta_i$,Var $= 1/\eta_i^2$.

The deterministic constraint becomes

$\sum_{j=1}^{n} a_{ij}\, x_j \le -\frac{\ln(1-\zeta_i)}{\eta_i}$, and the corresponding deterministic problem is (P2) Maximize $F^{(k)}(x) = SC(\sum_{j=1}^{n}(\tilde{c}_j^{(k)})_{QROF})x_j$

subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le -\frac{\ln(1-\zeta_i)}{\eta_i}, x_j \ge 0.$$

Gamma Distribution: If b_i follows a Gamma distribution follows with shape α and scale ω_i, its PDF is $\chi(b_i) = \frac{\omega_i^{\alpha} b_i^{\alpha-1} e^{-\omega_i b_i}}{\Gamma(\alpha)}, b_i \ge 0, \mu = \frac{\alpha}{\omega_i}$,Var $= \frac{\alpha}{\omega_i^2}$.

The probabilistic constraint can be transformed via the incomplete gamma function and appropriate substitution. The deterministic equivalent becomes

(P3) Maximize $F^{(k)}(x) = SC(\sum_{j=1}^{n}(\tilde{c}_j^{(k)})_{QROF})x_j$, subject to a Gamma-based inequality constraint, $x_j \ge 0$.

Log-Normal Distribution: If b_i is log-normally distributed, then $\ln(b_i)$ is normally distributed with mean $\mu_i$and variance σ_i^2. The expected value and variance of b_i are $E(b_i) = e^{\mu_i + \sigma_i^2/2}$,Var$(b_i) = e^{2\mu_i + \sigma_i^2}(e^{\sigma_i^2} - 1)$.

The deterministic equivalent of the probabilistic constraint is $\sum_{j=1}^{n} a_{ij}\, x_j \le e^{\mu_i + M_{\Phi_i}\sigma_i}$, M_{Φ_i} is the standard normal quintile corresponding to probability $1 - \zeta_i$. Thus, the deterministic problem becomes (P4) Maximize $F^{(k)}(x) = SC(\sum_{j=1}^{n}(\tilde{c}_j^{(k)})_{QROF})x_j$ subject to $\sum_{j=1}^{n} a_{ij}\, x_j \le e^{\mu_i + M_{\Phi_i}\sigma_i}, x_j \ge 0$.

4. Membership Functions

To solve the deterministic problems (P1)–(P4) using a fuzzy compromise approach, several MFs can be employed to represent the degree of satisfaction of each objective.

4.1 Linear Membership Function

For the k-th objective function, the linear MF (LMF) is defined as [30]:

$$\upsilon_k(x) = \begin{cases} 0, & F^{(k)} \le L^k, \\ \dfrac{F^{(k)} - L^k}{U^k - L^k}, & L^k < F^{(k)} < U^k, \\ 1, & F^{(k)} \ge U^k, \end{cases}$$

where $\upsilon_k(x)$denotes the membership value of the k-th objective, and L^k and U^k are the lower and upper bounds of $F^{(k)}$, respectively.

4.2 Hyperbolic Membership Function

The hyperbolic MF for the k-th objective is expressed as [29]:

$\upsilon_k(x) = \frac{1}{2}\tanh\left[a^k\left(F^{(k)} - \frac{U^k - L^k}{2}\right)\right] + \frac{1}{2}$, where $a^k > 0$ is a scaling parameter, typically set as $a^k = \frac{6}{U^k - L^k}$. Here, L^k and U^k represent the lower and upper bounds of the k-th objective, as before.

4.3 Nonlinear MF Using the Product Operator

When multiple objectives are considered, an overall satisfaction level can be calculated by aggregating the individual membership values using a product operator [30]:

$$\upsilon(x) = \prod_{k=1}^{r} \upsilon_k\,(x),$$

where r is the total number of objectives, and $\upsilon_k(x)$ is the MF for the k-th objective, which can be defined as:

$$\upsilon_k(x) = \frac{F^{(k)} - L^k}{U^k - L^k}, L^k < F^{(k)} < U^k.$$

This approach ensures that the overall membership value reflects a compromise among all objectives.

5. Solution Technique

This section presents a systematic procedure to solve the stochastic q-ROF-MOLPP:

Step 1: Formulate the MOLP problem where the coefficients of the objective functions are represented by triangular q-ROFNs, and only the right-hand side (RHS) constraints $b_i$are treated as probabilistic variables.

Step 2: Transform the stochastic q-ROF-MOLPP into a deterministic equivalent by combining the score function for fuzzy coefficients with chance-constrained programming for the probabilistic RHS. This converts the original problem into a crisp MOLP.

Step 3: Solve each objective function individually with respect to the deterministic constraints to determine the ideal solutions. The optimal value of each objective function corresponding to these solutions can be computed using optimization software such as LINGO 20.0.

Step 4: Identify the lower and upper bounds, L^k and U^k, for each objective function based on the ideal solutions obtained.

Step 5: Apply the MF described in Section 4 to formulate a fuzzy compromise programming model. The specific formulations are as follows:

Using Linear MF:

$$\text{Maximize } \xi$$
$$\text{subject to } \xi \leq \frac{F^{(k)} - L^k}{U^k - L^k}, \sum_{i=1}^{n} a_{ij}\, x_j \leq b_i, x_j \geq 0, \xi \geq 0, i = 1,2, \dots, m,$$

Using Nonlinear MF with Product Operator:

$$\text{Maximize } \prod_{k=1}^{r} v_k\,(x)$$
$$\text{subject to } \sum_{i=1}^{n} a_{ij}\, x_j \leq b_i, x_j \geq 0, i = 1,2, \dots, m,$$

Using Hyperbolic MF:

$$\text{Maximize } \xi$$
$$\text{subject to } \xi - \frac{1}{2}\tanh\,[a^k(F^{(k)} - \frac{U^k - L^k}{2})] \leq \frac{1}{2},$$
$$\sum_{i=1}^{n} a_{ij}\, x_j \leq b_i, x_j \geq 0, \xi \geq 0, i = 1,2, \dots, m.$$

Step 6: Construct the corresponding fuzzy compromise problem based on the selected MF (linear, nonlinear-product, or hyperbolic).

Step 7: Solve the constructed problem using LINGO 20.0 to obtain the compromise solution for all objectives.

6. Numerical Examples

To demonstrate the applicability of the proposed approach, two numerical examples are considered.

6.1 Example 1: Exponentially Distributed $\boldsymbol{b_i}$

Consider the following two-objective q-ROF-MOLP:

$$\text{Max } \tilde{F}_{QRF}^{(1)} = \langle (15,18,21); (0.68,0.2)^q \rangle x_1 \oplus \langle (9,12,15); (0.52,0.4)^q \rangle x_2,$$
$$\text{Max } \tilde{F}_{QRF}^{(2)} = \langle (3,5,7); (0.82,0.12)^q \rangle x_1 \oplus \langle (12,14,16); (0.6,0.3)^q \rangle x_2,$$
$$\text{Subject to } \begin{cases} \Pr(x_1 + x_2 \leq b_1) \geq 0.94, \\ \Pr(4x_1 + 3x_2 \leq b_2) \geq 0.93, \\ \Pr(2x_1 + x_2 \leq b_3) \geq 0.91, \\ x_1, x_2 \geq 0, \end{cases}$$

with parameters: $\mu(b_1) = 7, \mu(b_2) = 9, \mu(b_3) = 8$, and $\varsigma_1 = 0.06, \varsigma_2 = 0.07, \varsigma_3 = 0.09$.

Step 1: Let $q = 3$and convert the problem into a deterministic MOLP:

$$\text{Max } F^{(1)} = 11.76x_1 + 6.46x_2,$$
$$\text{Max } F^{(2)} = 3.18x_1 + 8.32x_2,$$
$$\text{Subject to } \begin{cases} x_1 + x_2 \leq 0.43, \\ 4x_1 + 3x_2 \leq 0.65, \\ 2x_1 + x_2 \leq 0.76, \\ x_1, x_2 \geq 0. \end{cases}$$

Step 2: Determine the ideal solutions for each objective:

$\bar{F}^{(1)} \quad = 1.911 \text{at} \bar{x}_1 = 0.1625, \bar{x}_2 = 0,$
$\bar{F}^{(2)} \quad = 1.803 \text{at} \bar{x}_1 = 0, \bar{x}_2 = 0.2166.$

Step 3: Apply the linear MF to construct the compromise model:

$$\text{Max } \xi$$
$$\text{Subject to } \begin{cases} 107.89x_1 + 59.27x_2 - \xi \geq 9.93, \\ 2.47x_1 - 6.47x_2 - \xi \geq 0.40, \\ x_1 + x_2 \leq 0.43, \\ 4x_1 + 3x_2 \leq 0.65, \\ 2x_1 + x_2 \leq 0.76, \\ x_1, x_2, \xi \geq 0. \end{cases}$$

Step 4: The compromise solution obtained is:
$\xi = 0.6621, x_1 = 0.43, x_2 = 0, F^{(1)} = 5.0568, F^{(2)} = 1.3674.$

6.2 Example 2: Uniformly Distributed b_i

Consider the problem:

$$\text{Max } \tilde{F}_{QRF}^{(1)} = \langle (4,5,6); (0.7,0.2)^q \rangle x_1 \oplus \langle (4.5,5,6.5); (0.5,0.3)^q \rangle x_2,$$
$$\text{Max } \tilde{F}_{QRF}^{(2)} = \langle (0.2,1,3.2); (0.8,0.18)^q \rangle x_1 \oplus \langle (9,12,15); (0.52,0.4)^q \rangle x_2,$$
$$\text{Subject to } \begin{cases} \Pr(x_1 + x_2 \leq 7.6) \geq 1 - \varsigma_1, \\ \Pr(5x_1 + 3x_2 \leq 7.8) \geq 1 - \varsigma_2, \\ \Pr(x_1 + 4x_2 \leq 7.2) \geq 1 - \varsigma_3, \\ x_1, x_2 \geq 0, \end{cases}$$

with parameters: $\mu(b_1) = \mu(b_2) = \mu(b_3) = 6, \text{Var}(b_1) = \text{Var}(b_2) = \text{Var}(b_3) = 4,$ and $\varsigma_1 = 0.9, \varsigma_2 = 0.95, \varsigma_3 = 0.8.$

Step 1: Let $q = 3$and convert to deterministic MOLP:

$$\text{Max } F^{(1)} = x_1 + 3x_2,$$
$$\text{Max } F^{(2)} = x_1 + 5x_2,$$
$$\text{Subject to } \begin{cases} 2x_1 + x_2 \leq 0.76, \\ 5x_1 + 3x_2 \leq 7.8, \\ x_1 + 4x_2 \leq 0.72, \\ x_1, x_2 \geq 0. \end{cases}$$

Step 2: Compute the ideal solutions:
$\bar{F}^{(1)} \quad = 1.911 \text{at} \bar{x}_1 = 0.1625, \bar{x}_2 = 0,$
$\bar{F}^{(2)} \quad = 1.803 \text{at} \bar{x}_1 = 0, \bar{x}_2 = 0.2166.$

Step 3: Construct the compromise problem using the linear MF:

$$\text{Max } \xi$$

Subject to constraints identical to Example 1: $x_1 + x_2 \leq 0.43, \dots, x_1, x_2, \xi \geq 0.$

Step 4: The compromise solution is (Table 1):

$$\xi = 0.6621, \bar{x}_1 = 0.43, \bar{x}_2 = 0, \bar{F}^{(1)} = 5.0568, \bar{F}^{(2)} = 9.3674.$$

Table 1
Comparison of Examples 1,2 with the existing methods

Example 1	Existing method	Example 2	Existing method
Sinha *et al.,* [31]	$F^{(1)} = 1.6325$ $F^{(2)} = 1.071$	Sinha *et al.,* [31]	$F^{(1)} = 5.541$ $F^{(2)} = 9.00$
Our approach	$F^{(1)} = 5.0568$ $F^{(2)} = 1.3674$	Our approach	$F^{(1)} = 5.5568$ $F^{(2)} = 9.012$

7. Conclusion and Future works

This study investigates a MOLP problem in which the coefficients of the objective functions are represented by q-ROFN, while the right-hand side parameters (b_i) are treated as probabilistic variables with known means and variances. To handle both fuzziness and randomness, the score function is applied to the q-ROFNs, and various probability distributions, including Gamma, log-normal, and exponential, are used to model the stochastic constraints. The resulting MOLP problem is then transformed into an equivalent deterministic MOLP. A fuzzy compromise approach, based on Zimmermann's methodology and the linear MF, is employed to incorporate decision-makers' preferences and derive a compromise solution. Numerical examples are provided to demonstrate the practical applicability of the proposed approach. Future research could extend this framework to other fuzzy environments, such as intuitionistic, Pythagorean, or spherical fuzzy sets, explore alternative ranking functions, and integrate recent DM techniques to further enhance the model.

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Conflicts of Interest

The authors declare no conflicts of interest.

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