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Interpretable Robust Multicriteria Ranking with TODIM in Generalized Orthopair Fuzzy Settings

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ABSTRACT

The endeavor to align TODIM (an acronym in Portuguese for interactive and multicriteria decision making) with prospect theory has given rise to the development of several variant methods, including power TODIM, exponential TODIM, and logarithmic TODIM. However, these existing methods fail to address high-order uncertainty within generalized orthopair fuzzy environments. To overcome this limitation, we propose an interpretable robust TODIM approach tailored for generalized orthopair fuzzy settings. First, we extend these TODIM methods to accommodate generalized orthopair fuzzy settings, integrating them into a unified framework. Second, we introduce a set of robustness analysis measures for the extended TODIM method, accounting for simultaneous uncertainty in criteria weights, value function coefficients, and the membership and non-membership degrees of generalized orthopair fuzzy sets. Third, we develop a programming model to determine representative criteria weights based on these robustness analysis measures, followed by an approach to recommend an interpretable and robust ranking within the extended TODIM framework. Finally, we present an illustrative example to demonstrate the application of this interpretable and robust TODIM approach, accompanied by a comparative analysis to highlight its advantages.

1. Introduction

Multiple criteria decision analysis (MCDA) is a frequently encountered process in human activities, involving the selection, ranking, or sorting of alternatives based on multiple and often conflicting criteria [1, 2]. Traditional MCDA methods can be classified into value-based methods and outranking-based methods. The AHP (Analytic Hierarchy Process) and ANP (Analytic Network Process) methods

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are examples of the former, while ELECTRE (an acronym in French of elimination and choice translating reality) methods belong to the latter category. However, TODIM (an acronym in Portuguese of interactive and multicriteria decision making), which is originally proposed by Gomes & Lima [3, 4], can be viewed as a distinct category, as it combines elements of the multiattribute utility theory from the AHP method with features of the ELECTRE methods [5]. Another distinguishing feature of the TODIM method is its incorporation of psychological behavioral factors of decision-makers (DMs) in situations involving risk and uncertainty, grounded in the principles of prospect theory [6]. To date, the TODIM method has been successfully applied across a wide range of industries, including healthcare, real estate, energy, manufacturing, and service sectors.

The basic principle of TODIM depends on a value function (or Phi function) that calculates the global dominance degree of one alternative over others across all criteria under consideration. A key step in this process involves calculating the dominance degree of one alternative over another for a given criterion using the Phi function. These individual dominance degrees are subsequently aggregated to determine the global dominance degrees for each alternative, which are then used to rank alternatives. In the original formulation of the TODIM method [3, 4], the Phi function was constructed using a square root structure, incorporating the weight of the considered criterion within this structure. However, this construction could yield ambiguous or equivocal results in certain scenarios [7]. Subsequently, Lee and Shih [8] generalized the original square root structure by introducing a power function, retaining the weight component within the power function. This generalization, however, resulted in a structure that was not fully aligned with the value function of prospect theory. To address this, Llamazares [9] proposed a revised version of the power Phi function, where the weight component is no longer integrated into the power function. For simplicity, this variant of TODIM is referred to as power TODIM (PowTODIM) in this paper. Recently, Leoneti and Gomes [10] proposed new variants of TODIM, substituting the power function in PowTODIM with exponential or logarithmic functions. For simplicity, these derived variants of TODIM are referred to as exponential TODIM (ExpTODIM) and logarithmic TODIM (LogTODIM), respectively. It should be noted that the value functions in PowTODIM, ExpTODIM, and LogTODIM all exhibit an S-shaped curve, each adhering to the value function defined in prospect theory. A key distinction among these value functions lies in their bounds: the value function in ExpTODIM is limited to a range between -λ and 1, whereas the value functions in PowTODIM and LogTODIM are unbounded.

The accurate characterization of DMs' value judgments and preferences is fundamental to MCDA. In all the TODIM methods discussed earlier, the evaluation of an alternative for a given criterion is based on binary logic. As a result, these methods are limited to accepting evaluations in the form of crisp numbers and are ill-equipped to address scenarios where DMs can only provide uncertain or imprecise evaluations. To overcome this limitation, various extensions of the original TODIM method have been developed to accommodate evaluations expressed as fuzzy sets [11], intuitionistic fuzzy sets (IFSs) [7], Pythagorean fuzzy sets (PFSs) [12], generalized orthopair fuzzy sets (GOFSs) [13], interval type-2 fuzzy sets [14], hybrid data [15], and more. Among these evaluation frameworks, GOFSs stand out as a promising tool due to their flexibility in characterizing the degrees of support (membership) and opposition (non-membership) of an alternative relative to a given criterion. In GOFSs, the sum of the qth power of the membership degree and the non-membership degree is constrained to be less than or equal to one. When q equals 1, GOFSs reduce to IFSs; when q equals 2, they simplify to PFSs. As the parameter q (referred to as the rung) increases, the range of acceptable orthopairs expands, granting DMs greater freedom to articulate their preferences. In this context, GOFSs are often termed ***q***-rung orthopair fuzzy sets (q-ROFSs) when fixed at a specific rung q. The notable advantages of q-ROFSs have attracted considerable interest from numerous researchers, sparking extensive investigations in the field of MCDA [16]. Our paper is motivated by the following factors:

(1) Few efforts have been made to extend the PowTODIM, ExpTODIM, and LogTODIM methods to

generalized orthopair fuzzy environments. A notable exception is the work by Liu *et al.* [17], who extended the LogTODIM method to accommodate q-ROFSs. However, their approach relies solely on a distance measure to assess the absolute differences between q-ROFSs, neglecting their relative differences. This oversight can distort decision outcomes, especially in cases involving extreme evaluations. Furthermore, existing studies fail to address high-order uncertainty in q-ROFSs when applying the PowTODIM, ExpTODIM, and LogTODIM methods. Although Zhang *et al.* [18] investigated the uncertainty inherent in membership and non-membership degrees of q-ROFSs, they did not explicitly incorporate the psychological behavior of DMs, which is a fundamental aspect of these TODIM methods.

(2) The existing approaches inadequately address simultaneous uncertainty in criteria weights, value function coefficients, and the membership and non-membership degrees of q-ROFSs. While SMAA-TODIM [19] and SMAA-ExpTODIM [20] have been developed to account for simultaneous uncertainty in criteria measurements, criteria weights, and value function coefficients, they are not designed to accommodate q-ROFSs. Additionally, these methods do not provide robustness analysis for the PowTODIM and LogTODIM variants.

(3) The final ranking of alternatives produced by SMAA-TODIM [19] and SMAA-ExpTODIM [20] lack interpretability for DMs, as these SMAA-based methods fail to provide representative criteria weights underlying the final rankings.

The aim of this paper is to propose an interpretable robust TODIM approach within the context of generalized orthopair fuzzy environments, addressing the limitations outlined previously. The originality of our study can be summarized by highlighting the following key aspects:

(1) We extend the PowTODIM, ExpTODIM, and LogTODIM methods to generalized orthopair fuzzy settings by introducing a grey relational coefficient-based relative closeness degree, unifying these three methods within a single framework.

(2) We introduce a set of robustness analysis measures for the extended PowTODIM, ExpTODIM, and LogTODIM methods, rooted in the principles of SMAA (Stochastic Multiobjective Acceptability Analysis). This analysis effectively handles simultaneous uncertainty in criteria weights, value function coefficients, and the high-order uncertainty inherent in q-ROFSs.

(3) We develop a mathematical programming model to determine representative criteria weights by leveraging measures derived from the robustness analysis.

(4) We propose an approach to generate an interpretable robust ranking from the extended TODIM framework.

The paper is structured as follows. Section 2 offers a concise overview of the foundational concepts underlying the TODIM methods and q-rung orthopair fuzzy sets. Section 3 details the extension of the PowTODIM, ExpTODIM, and LogTODIM methods to generalized orthopair fuzzy environments. In Section 4, we introduce several robustness analysis measures for the extended TODIM method and propose a method for determining representative criteria weights to facilitate the interpretation of the resulting robust ranking. Section 5 provides an illustrative example and a comparison analysis to demonstrate the application and advantages of the proposed approach. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, we first review the TODIM methods and then briefly discuss some concepts related to q-ROFSs.

2.1 The TODIM methods

TODIM is a MCDA method inspired by prospect theory [6] and initially developed by Gomes and Lima [3, 4] to address ranking and choice problems. Since its introduction, it has evolved into multiple variants, most of which are based on three types of Phi functions, as discussed in Leoneti *et al.* [10]. These developments have given rise to the PowTODIM, ExpTODIM, LogTODIM methods. The application of these methods typically involves the following steps.

Step 1. Normalize the original decision matrix $\boldsymbol{x} = [x_{ij}]_{m\times n}$ to a standard decision matrix $\boldsymbol{y} = [y_{ij}]_{m\times n}$ using a linear technique, where x_{ij} represents the measurement of an alternative a_i ($i \in M = \{1, 2, ..., m\}$) in terms of a criterion g_j ($j \in N = \{1, 2, ..., n\}$).

Step 2. For each $i, h \in M$ and $j \in N$, calculate the dominance degree of alternative a_i over a_h for criterion g_j using Phi function $\varphi_j(a_i, a_h)$:

$$\varphi_j(a_i, a_h) = w_j \psi_j(a_i, a_h) \tag{1}$$

where w_j is the weight of criterion g_j such that $w_j \geq 0 (j = 1, 2, \dots, n)$ and $\sum_{j=1}^{n} w_j = 1$, and $\psi_j(a_i, a_h)$ is a value function. In particular, the value function in the TODIM methods can take the power, exponential, and logarithmic formulations:

- The power value function

$$\psi_j^{pow}(a_i, a_h) = \begin{cases} (y_{ij} - y_{hj})^{\alpha}, & \text{if } (y_{ij} - y_{hj}) > 0 \\ 0, & \text{if } (y_{ij} - y_{hj}) = 0 \\ -\lambda(y_{hj} - y_{ij})^{\beta}, & \text{if } (y_{ij} - y_{hj}) < 0 \end{cases} \tag{2}$$

- The exponential value function

$$\psi_j^{exp}(a_i, a_h) = \begin{cases} 1 - 10^{-\rho(y_{ij} - y_{hj})}, & \text{if } (y_{ij} - y_{hj}) > 0 \\ 0, & \text{if } (y_{ij} - y_{hj}) = 0 \\ -\lambda(1 - 10^{\rho(y_{hj} - y_{ij})}), & \text{if } (y_{ij} - y_{hj}) < 0 \end{cases} \tag{3}$$

- The logarithmic value function

$$\psi_j^{log}(a_i, a_h) = \begin{cases} \log(1 + 10\rho(y_{ij} - y_{hj})), & \text{if } (y_{ij} - y_{hj}) > 0 \\ 0, & \text{if } (y_{ij} - y_{hj}) = 0 \\ -\lambda \log(1 + 10\rho(y_{hj} - y_{ij})), & \text{if } (y_{ij} - y_{hj}) < 0 \end{cases} \tag{4}$$

such that the value function coefficients satisfy $0 < \alpha, \beta \leq 1$, $\lambda > 1$, $\rho \in \{1, 2, 3, 4, 5\}$.

Step 3. For each $i, h \in M$, calculate the dominance ratio of alternative a_i over a_h by Eq. (5):

$$\delta_i(a_i, a_h) = \sum_{j=1}^{n} \varphi_j(a_i, a_h). \tag{5}$$

Step 4. For each $i \in M$, calculate the global dominance degree of alternative a_i based on the sum of the dominance ratio and normalize these degrees between 0 and 1 by Eq. (6):

$$\pi(a_i) = \frac{\sum_{h=1}^{m} \delta_i(a_i, a_h) - \min_{i\in M} \sum_{h=1}^{m} \delta_i(a_i, a_h)}{\max_{i\in M} \sum_{h=1}^{m} \delta_i(a_i, a_h) - \min_{i\in M} \sum_{h=1}^{m} \delta_i(a_i, a_h)} \tag{6}$$

Step 5. Rank the alternatives according to $\pi(a_i)(i = 1, 2, \dots, m)$ in descending order. A larger value of $\pi(a_i)$ corresponds to a better alternative.

2.2 Some concepts on q-ROFSs

In the following, we review the basic concept of the q-ROFS and its associated distance measure.
Definition 1 [21]. Let X be a nonempty fixed set, then a q-ROFS Q on X can be expressed as follows:

$$Q = \{< x, \alpha_Q(x), \beta_Q(x) > | x \in X\}, \tag{7}$$

where $\alpha_Q(x) : X \to [0,1]$ and $\beta_Q(x) : X \to [0,1]$ denote the membership degree and the non-membership degree of x to Q, respectively, which satisfy the following condition for each $x \in X$: $0 \leq (\alpha_Q(x))^q + (\beta_Q(x))^q \leq 1$ ($q \geq 1$). The degree of indeterminacy of element $x \in X$ is given as $\pi_Q(x) = (1 - (\alpha_Q(x))^q - (\beta_Q(x))^q)^{1/q}$.

For convenience, $\langle \alpha_{\tilde{y}}(x), \beta_{\tilde{y}}(x) \rangle$ can be called a q-rung orthopair fuzzy number (q-ROFN) [22], denoted by $\tilde{y} = \langle \alpha_{\tilde{y}}, \beta_{\tilde{y}} \rangle$.
Definition 2 [23]. Let $\tilde{y}_1 = \langle \alpha_1, \beta_1 \rangle$ and $\tilde{y}_2 = \langle \alpha_2, \beta_2 \rangle$ be two q-ROFNs, the Minkowski distance between $\tilde{y}_1$ and $\tilde{y}_2$ is defined as

$$d^p(\tilde{y}_1, \tilde{y}_2) = \left(\frac{1}{2} |\alpha_1 - \alpha_2|^p + \frac{1}{2} |\beta_1 - \beta_2|^p \right)^{1/p}, \tag{8}$$

where $p \geq 1$. Especially, the Minkowski distance reduces to Hamming distance and Euclidean distance when $p = 1$ and $p = 2$, respectively.

3. Extended TODIM to accommodate *q*-rung orthopair fuzzy settings

The TODIM methods traditionally accommodate decision matrix with precise values, neglecting situations where evaluations of a given alternative across all considered criteria are represented by q-ROFNs. In this section, we aim to extend the TODIM methods to handle *q*-rung orthopair fuzzy information, which we will refer to as the qROF-ETODIM method throughout the rest of the paper.

Consider a MCDM problem that a set of m alternatives $(a_i, i \in \{1, 2, \ldots, m\})$ are to be evaluated or ranked in terms of n criteria $(g_j, j \in \{1, 2, \ldots, n\})$. Assume that the measurement of alternative a_i w.r.t criterion g_j is represented by a q-ROFN, denoted as $\langle \mu_{ij}, \nu_{ij} \rangle$. The orthopair $\langle \mu_{ij}, \nu_{ij} \rangle$ captures the degrees of membership and non-membership, respectively, indicating the extent to which alternative a_i supports and opposes criterion g_j. The considered criteria are assumed to be independent, and the weights for them are denoted by $\boldsymbol{w} = [w_j]_{1 \times n}$, such that $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$. The specific steps of the qROF-ETODIM method are presented as follows.

Step 1. Normalize the *q*-rung orthopair fuzzy decision matrix $\boldsymbol{x} = [x_{ij}]_{m \times n} = (\langle \mu_{ij}, \nu_{ij} \rangle)_{m \times n}$ to matrix $\boldsymbol{r} = (\tilde{r}_{ij})_{m \times n} = \langle \bar{\mu}_{ij}, \bar{\nu}_{ij} \rangle_{m \times n}$ by transforming cost-type criteria into benefit-type criteria, such that $\langle \bar{\mu}_{ij}, \bar{\nu}_{ij} \rangle = \langle \mu_{ij}, \nu_{ij} \rangle$ for benefit criteria and $\langle \bar{\mu}_{ij}, \bar{\nu}_{ij} \rangle = \langle \nu_{ij}, \mu_{ij} \rangle$ for cost criteria.

Step 2. Transform the normalized q-rung orthopair fuzzy decision matrix into a grey relational coefficient-based relative closeness degree matrix.

For a normalized q-rung orthopair fuzzy decision matrix $\boldsymbol{r} = (\tilde{r}_{ij})_{m \times n}$ with $\tilde{r}_{ij} = \langle \bar{\mu}_{ij}, \bar{\nu}_{ij} \rangle$, let $r_i^+ = \langle 1, 0 \rangle$ and $r_i^- = \langle 0, 1 \rangle$ be the absolute positive ideal point (APIS) and absolute negative ideal point (ANIS) under criterion g_j, respectively. Following the principle of grey relational analysis, we define the grey relational coefficient (GRC) between the alternative a_i and the APIS in terms of criterion g_j as follows:

$$o^+(\tilde{r}_{ij}) = \frac{\min_i \min_j d^p(\tilde{r}_{ij}, r_j^+) + \chi \max_i \max_j d^p(\tilde{r}_{ij}, r_j^+)}{d^p(\tilde{r}_{ij}, r_j^+) + \chi \max_i \max_j d^p(\tilde{r}_{ij}, r_j^+)}, \tag{9}$$

and the GRC between the alternative a_i and the ANIS in terms of criterion g_j as follows:

$$o^-(\tilde{r}_{ij}) = \frac{\min_i \min_j d^p(\tilde{r}_{ij}, r_j^-) + \chi \max_i \max_j d^p(\tilde{r}_{ij}, r_j^-)}{d^p(\tilde{r}_{ij}, r_j^-) + \chi \max_i \max_j d^p(\tilde{r}_{ij}, r_j^-)}, \tag{10}$$

where χ in Eqs. (9) and (10) is a distinguishing coefficient such that $0 < \chi \leq 1$, and the distance between different q-ROFNs can be measured using Eq. (8).

The GRC $o^+(\tilde{r}_{ij})$ $(j = 1, 2, \ldots, n)$ represents the degree of correlation between the alternative a_i and the APIS. A higher value of $o^+(\tilde{r}_{ij})$ $(j = 1, 2, \ldots, n)$ indicates a stronger correlation with APIS and suggests that the alternative is more favorable. On the other hand, the GRC $o^-(\tilde{r}_{ij})$ $(j = 1, 2, \ldots, n)$ represents the degree of correlation between the alternative a_i and the ANIS. A higher value of $o^-(\tilde{r}_{ij})$ $(j = 1, 2, \ldots, n)$ indicates a stronger correlation with ANIS and suggests that the alternative is less favorable.

For a given normalized decision matrix $(\tilde{r}_{ij})_{m\times n}$, we can formulate two GRC matrices, $\Delta^+ = [o^+(\tilde{r}_{ij})]_{m\times n}$ and $\Delta^- = [o^-(\tilde{r}_{ij})]_{m\times n}$, based on Eqs. (9) and (10), respectively. To comprehensively evaluate the performance of the considered alternative under a given criterion, inspired by the TOPSIS method, the individual relative closeness degree $\eta(\tilde{r}_{ij})$ in terms of GRCs can be defined as:

$$\eta(\tilde{r}_{ij}) = \frac{o^+(\tilde{r}_{ij})}{o^+(\tilde{r}_{ij}) + o^-(\tilde{r}_{ij})}, \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \tag{11}$$

where the higher the value of $\eta(\tilde{r}_{ij})$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ indicates a better performance of the alternative.

Step 3. Choose a value function formulation from the following three types and determine the coefficients of the selected value function.

- The power value function

$$\psi_j^{pow}(a_i, a_h) = \begin{cases} (\eta(\tilde{r}_{ij}) - \eta(\tilde{r}_{hj}))^\alpha, & \text{if } \eta(\tilde{r}_{ij}) > \eta(\tilde{r}_{hj}) \\ 0, & \text{if } \eta(\tilde{r}_{ij}) = \eta(\tilde{r}_{hj}) \\ -\lambda(\eta(\tilde{r}_{hj}) - \eta(\tilde{r}_{ij}))^\beta, & \text{if } \eta(\tilde{r}_{ij}) < \eta(\tilde{r}_{hj}) \end{cases} \tag{12}$$

- The exponential value function

$$\psi_j^{exp}(a_i, a_h) = \begin{cases} 1 - 10^{-\rho(\eta(\tilde{r}_{ij}) - \eta(\tilde{r}_{hj}))}, & \text{if } \eta(\tilde{r}_{ij}) > \eta(\tilde{r}_{hj}) \\ 0, & \text{if } \eta(\tilde{r}_{ij}) = \eta(\tilde{r}_{hj}) \\ -\lambda(1 - 10^{-\rho(\eta(\tilde{r}_{hj}) - \eta(\tilde{r}_{ij}))}), & \text{if } \eta(\tilde{r}_{ij}) < \eta(\tilde{r}_{hj}) \end{cases} \tag{13}$$

- The logarithmic value function

$$\psi_j^{log}(a_i, a_h) = \begin{cases} \log(1 + 10\rho(\eta(\tilde{r}_{ij}) - \eta(\tilde{r}_{hj}))), & \text{if } \eta(\tilde{r}_{ij}) > \eta(\tilde{r}_{hj}) \\ 0, & \text{if } \eta(\tilde{r}_{ij}) = \eta(\tilde{r}_{hj}) \\ -\lambda \log(1 + 10\rho(\eta(\tilde{r}_{hj}) - \eta(\tilde{r}_{ij}))), & \text{if } \eta(\tilde{r}_{ij}) < \eta(\tilde{r}_{hj}) \end{cases} \tag{14}$$

Note that in the extended TODIM method, we define the parameter ρ to take values within the real number domain $[1, 5]$ to accommodate group decision-making scenarios. In such contexts, each DM can express his/her individual preference for the value of ρ, and the collective opinion for the value of ρ can be any real number between 1 and 5.

Step 4. For each $i, h \in M$, calculate the dominance ratio of alternative a_i over a_h by Eq. (15):

$$\delta_i(a_i, a_h) = \begin{cases} \sum_{j=1}^{n} w_j \psi_j^{pow}(a_i, a_h), & \text{if power value function is considered} \\ \sum_{j=1}^{n} w_j \psi_j^{exp}(a_i, a_h), & \text{if exponential value function is considered} \\ \sum_{j=1}^{n} w_j \psi_j^{\log}(a_i, a_h), & \text{if logarithmic value function is considered} \end{cases} \tag{15}$$

Step 5. For each $i \in M$, calculate the performance of alternative a_i based on the sum of the dominance ratio by Eq. (16):

$$\pi(a_i) = \sum_{h=1}^{m} \delta_i(a_i, a_h). \tag{16}$$

Alternatively, normalize the values of $\pi(a_i)$ ($i = 1, 2, \ldots, m$) between 0 and 1 by Eq. (17):

$$\bar{\pi}(a_i) = \frac{\pi(a_i) - \min_{i \in M} \pi(a_i)}{\max_{i \in M} \pi(a_i) - \min_{i \in M} \pi(a_i)}. \tag{17}$$

Step 6. Rank the alternatives according to $\pi(a_i)$ or $\bar{\pi}(a_i)$ ($i = 1, 2, \ldots, m$) in descending order. A larger value of $\pi(a_i)$ or $\bar{\pi}(a_i)$ corresponds to a better alternative.

The implementation of the qROF-ETODIM method involves several parameters, such as criteria weights and coefficients of the chosen value function. If the DMs can reach a consensus on these parameters and obtain precise values, the ranking of alternatives can be directly determined using the outlined steps. However, if the group can only provide partial preference information or cannot provide any preference information—and even the membership and non-membership preferences of the alternatives cannot be accurately determined—robustness analysis for the qROF-ETODIM method becomes necessary.

4. Interpretable robust ranking for the extended TODIM

The SMAA framework provides a robust approach for addressing uncertainty in model parameters. For instance, SMAA-TODIM and SMAA-ExpTODIM have been developed to manage uncertainty in the traditional TODIM and ExpTODIM methods, respectively. However, these approaches are not well-suited for handling q-rung orthopair fuzzy information. Furthermore, the DMs often struggle to interpret the robust rankings produced by SMAA-TODIM and SMAA-ExpTODIM. In this section, we apply SMAA principles to evaluate the robustness of ranking results derived from the qROF-ETODIM method when confronted with uncertain, imprecise, or incomplete information. Additionally, we propose a model to identify representative criteria weights based on robustness analysis measures, thereby enhancing the understanding and interpretation of the resulting robust ranking.

4.1 *Robustness analysis for the extended TODIM*

The qROF-ETODIM method can be regarded as a real-valued function, $\pi_i = \mathrm{T}(i, \boldsymbol{x}, \boldsymbol{w}, \boldsymbol{s})$, which assign a score π_i for alternative a_i by inputting a decision matrix $\boldsymbol{x}$ and some preference parameters, such as the criteria weighs $\boldsymbol{w}$, and the other parameters, denoted by a vector $\boldsymbol{s}$, such that $\boldsymbol{s} = [\alpha, \beta, \lambda, \chi, p]$ if the power value function is considered, and $\boldsymbol{s} = [\rho, \lambda, \chi, p]$ if the exponential or logarithmic value function is considered. When applying the qROF-ETODIM method, the uncertainty in measuring the extent to which an alternative supports or opposes a given criterion, that is, the membership degree and non-membership degree, can be respectively modeled by stochastic variables, $\boldsymbol{\xi}_{ij}^{\boldsymbol{\mu}}$, and $\boldsymbol{\xi}_{ij}^{\boldsymbol{\nu}}$, in feasible space $X \subseteq [0, 1]^{m \times n}$, whose joint probability density function (pdf) can be denoted by $f_X(\boldsymbol{\xi}^{\boldsymbol{\mu}})$

and $f_X(\boldsymbol{\xi}^{\boldsymbol{\nu}})$, respectively. Therefore, the pdf for criteria measurements can be formulated as a product $f_\Gamma(\boldsymbol{\xi}) = f_X(\boldsymbol{\xi}^{\boldsymbol{\mu}})f_X(\boldsymbol{\xi}^{\boldsymbol{\nu}})$, where Γ represents a feasible criterion measurement space such that $\Gamma = \{\boldsymbol{\xi} = \langle\boldsymbol{\xi}^{\boldsymbol{\mu}}, \boldsymbol{\xi}^{\boldsymbol{\nu}}\rangle : \boldsymbol{\xi}^{\boldsymbol{\mu}}, \boldsymbol{\xi}^{\boldsymbol{\nu}} \in X\}$. In cases where the DMs cannot provide any preference information on the preference parameters under consideration, the uncertainty in these parameters $\boldsymbol{w}, \boldsymbol{s}$ can be represented as independent variables within the spaces W, and S, respectively, such that

$$W = \left\{\boldsymbol{w} \in \mathbb{R}^n : w_j > 0, j = 1, 2, \ldots, n, \sum_{j=1}^{n} w_j = 1\right\},$$

$$S = \left\{\boldsymbol{s} \in \mathbb{R}^5 : 0 < \alpha \leq 1, 0 < \beta \leq 1, \lambda \geq 1, 0 < \chi \leq 1, p \geq 1\right\}$$

if the power value function is considered, or

$$S = \left\{\boldsymbol{s} \in \mathbb{R}^4 : 1 \leq \rho \leq 5, \lambda \geq 1, 0 < \chi \leq 1, p \geq 1\right\}$$

if the exponential or logarithmic value function is considered. Let the joint pdf of $\boldsymbol{w}$, and $\boldsymbol{s}$ be denoted by $f_W(\boldsymbol{w})$ and $f_S(\boldsymbol{s})$, respectively. If the DMs can provide some preference on $\boldsymbol{w}$, and $\boldsymbol{s}$, this information can be translated into corresponding constraints and incorporated into the respective feasible spaces, resulting in restricted feasible spaces. To avoid heavy notation, we use the same symbols for these restricted spaces in the rest of the paper.

Based on the principles of SMAA, we define a ranking function for the qROF-ETODIM method as follows:

$$\mathrm{rank}(i, \boldsymbol{x}, \boldsymbol{w}, \boldsymbol{s}) = 1 + \sum_{h \neq i} \kappa(\mathrm{T}(h, \boldsymbol{x}, \boldsymbol{w}, \boldsymbol{s}) > \mathrm{T}(i, \boldsymbol{x}, \boldsymbol{w}, \boldsymbol{s})), \tag{18}$$

where $\kappa(\mathrm{true}) = 1$ and $\kappa(\mathrm{false}) = 0$.

The SMAA-qROF-ETODIM method is therefore based on the definition of a set of measures, which is described as follows:

- The rank acceptability index (RAI), which represents the probability of an alternative a_i achieving the rth position in the final ranking, is defined by

$$b_i^r = \int_\Gamma f_\Gamma(\boldsymbol{\xi}) \int_{(\boldsymbol{w},\boldsymbol{s}) \in W \times S : \mathrm{rank}(i,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s}) = r} f_W(\boldsymbol{w}) f_S(\boldsymbol{s}) d\boldsymbol{w} d\boldsymbol{s} d\boldsymbol{\xi}. \tag{19}$$

- The strict pairwise winning index (SPWI), which represents the probability that alternative a_i is strictly preferred to a_k, is defined by

$$p_{ik}^S = \int_{(\boldsymbol{w},\boldsymbol{s}) \in W \times S : \mathrm{rank}(i,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s}) < \mathrm{rank}(k,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s})} f_W(\boldsymbol{w}) f_S(\boldsymbol{s}) \int_\Gamma f_\Gamma(\boldsymbol{\xi}) d\boldsymbol{\xi} d\boldsymbol{w} d\boldsymbol{s}. \tag{20}$$

- The weak pairwise winning index (WPWI), which represents the probability that alternative a_i is better than or indifferent to a_k, is defined by

$$p_{ik}^W = p_{ik}^S + \frac{1}{2} \int_{(\boldsymbol{w},\boldsymbol{s}) \in W \times S : \mathrm{rank}(i,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s}) = \mathrm{rank}(k,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s})} f_W(\boldsymbol{w}) f_S(\boldsymbol{s}) \int_\Gamma f_\Gamma(\boldsymbol{\xi}) d\boldsymbol{\xi} d\boldsymbol{w} d\boldsymbol{s}. \tag{21}$$

  It is evident that the WPWI satisfies the condition $p_{ik}^W + p_{ki}^W = 1$.

- The central weight vector (CWV), which represents a vector of criteria weights that reflect the preferences of a typical DM who ranks alternative a_i in the first place, is defined by

$$\boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{c}} = \frac{1}{b_i^1} \int_{\Gamma} f_{\Gamma}(\boldsymbol{\xi}) \int_{(\boldsymbol{w},\boldsymbol{s}) \in W \times S : \text{rank}(i,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s})=r} f_W(\boldsymbol{w}) f_S(\boldsymbol{s}) \boldsymbol{w} d\boldsymbol{w} d\boldsymbol{s} d\boldsymbol{\xi}. \tag{22}$$

- The central coefficient vector (CCV), which represents a vector of coefficient that reflect the preferences of a typical DM who ranks alternative a_i in the first place, is defined by

$$\boldsymbol{s}_{\boldsymbol{i}}^{\boldsymbol{c}} = \frac{1}{b_i^1} \int_{\Gamma} f_{\Gamma}(\boldsymbol{\xi}) \int_{(\boldsymbol{w},\boldsymbol{s}) \in Q \times W \times S : \text{rank}(i,\boldsymbol{x},\boldsymbol{w},\boldsymbol{s})=r} f_W(\boldsymbol{w}) f_S(\boldsymbol{s}) \boldsymbol{s} d\boldsymbol{w} d\boldsymbol{s} d\boldsymbol{\xi}. \tag{23}$$

The calculation of these robustness measures can be estimated using Monte Carlo simulation, as demonstrated in the work of Zhang *et al.* [18]. It is important to note that multiple rounds of robustness analysis can be conducted if the DMs are not satisfied with the calculated results of these measures. In such cases, the DMs can update their preference information regarding the considered parameters to refine the analysis.

4.2 *Determination of representative criteria weights from pairwise winning index*

The CWV as defined in Section 4.1 can provide value information on representative criteria weights for an alternative achieving the first position, rather than for the resulting robust ranking. To facilitate the interpretation of the final ranking, this section introduces a method for determining representative criteria weights based on WPWI. Therefore, it is reasonable to assume that the DMs have, through multiple rounds of robustness analysis, identified satisfactory values for all parameters except the criteria weights. However, in certain cases, uncertainty in $\boldsymbol{x}$ and $\boldsymbol{s}$ may still persist. To address this, we can use their expected values instead. Thus, we propose the following model for a general situation in the spirit of Arcidiacono *et al.* [24]:

$$\begin{aligned} &\vartheta^* = \max \vartheta \\ &s.t. \begin{cases} T(i, E(\boldsymbol{x}), \boldsymbol{w}, E(\boldsymbol{s})) - T(h, E(\boldsymbol{x}), \boldsymbol{w}, E(\boldsymbol{s})) \geq \vartheta(p_{ih}^W - 0.5), \forall (i,h) \in M \times M : p_{ih}^W \geq 0.5, i \neq h \\ \boldsymbol{w} \in W \end{cases} \end{aligned} \tag{24}$$

where $E(\boldsymbol{x}_{ij}) = \left\langle \int \xi_{ij}^{\mu} f_X(\xi_{ij}^{\mu}) d\xi_{ij}^{\mu}, \int \xi_{ij}^{\nu} f_X(\xi_{ij}^{\nu}) d\xi_{ij}^{\nu} \right\rangle$, $E(\boldsymbol{s}) = \int \boldsymbol{s} f_S(\boldsymbol{s}) d\boldsymbol{s}$, and the values of $p_{ih}^W (i, h = 1, 2, \ldots, m)$ are calculated under the settings of $E(\boldsymbol{x}_{ij})$ and $E(\boldsymbol{s})$.

The solutions of the model may fall into several distinct cases. (1) When $\vartheta^* > 0$, it indicates the existence of at least one set of weights that makes $a_i \succ a_h$ and $p_{ih}^W > 0.5$ equivalent for any pair of alternatives. (2) Conversely, when $\vartheta^* \leq 0$, it suggests the presence of at least one pair of alternatives where a_i is inferior to a_h despite $p_{ih}^W > 0.5$. In this scenario, the model can be understood as determining the criteria weights that minimize the deviation between the ranking of alternatives predicted by the extended TODIM method and that predicted by the WPWI. (3) If the model proves infeasible, it signals inconsistency in the preference information provided by the DMs. In such cases, the conflicting constraints can be identified using the approach proposed by Mousseau *et al.* [25], after which the solution reverts to either case (1) or (2). Thus, regardless of whether $\vartheta^* > 0$ or $\vartheta^* \leq 0$, it is always possible to derive a set of representative criteria weights. However, this set may not be unique. To address this, we denote the constraints in model (24) by Ω^{ϑ} and introduce an additional constraint $\vartheta = \vartheta^*$ to Ω^{ϑ}, forming a new constraint space Ω^{ϑ^*}. Ultimately, a unique set of criteria weights can be elicited by calculating the centroid of this new feasible space through Monte Carlo simulation.

4.3 Approach for deriving an interpretable robust ranking from the extended TODIM

In this section, we present an approach for deriving an interpretable robust ranking from the qROF-ETODIM method, building on the work outlined in Sections 4.1 and 4.2. The detailed steps are as follows:
Step 0. Identify an MCDA problem and collect evaluations of alternatives based on the criteria under consideration, utilizing the tool of q-ROFSs.
Step 1. Select an appropriate value function type from the available options—exponential value function, logarithmic value function, or power value function—and proceed to normalize the q-rung orthopair fuzzy decision matrix according to Section 3.
Step 2. The DMs are invited to provide (or update) preference information on certain parameters, such as criteria weights and value function coefficients. If these parameters are precisely determined, proceed to Step 5. Otherwise, continue to the next step.
Step 3. Generate samples from the feasible spaces of the considered parameters using the hit-and-run algorithm [26]. Based on these samples, estimate the robustness measures developed in Section 4.1. If the DMs are satisfied with the calculated results, proceed to the next step; otherwise, return to Step 2.
Step 4. Determine representative criteria weights by solving Model (24) and calculating the associated centroid of space Ω^{ϑ^*}. If the DMs are satisfied, proceed to the next step; otherwise, return to Step 2.
Step 5. Implement the extended TODIM method developed in Section 3 to derive a representative robust ranking of alternatives. If necessary, provide an interpretation of the results.

5. An illustrative example and comparative analysis

5.1 An illustrative example

To illustrate the application of the proposed method, an example is considered, involving the evaluation of five emergency response plans developed for COVID-19 based on four criteria: rescue capacity (g_1), response capacity (g_2), response cost (g_3), and potential impact of public opinion (g_4). The example is adapted from Zhang *et al.* [18]. Table 1 presents the normalized evaluations of alternatives across these four criteria.

Table 1
The normalized evaluations of alternatives under a given set of criteria

	g_1	g_2
a_1	$\langle[0.4, 0.5], [0.3, 0.4]\rangle$	$\langle[0.7, 0.8], [0.5, 0.6]\rangle$
a_2	$\langle[0.3, 0.4], [0.7, 0.8]\rangle$	$\langle[0.4, 0.5], [0.4, 0.5]\rangle$
a_3	$\langle[0.2, 0.3], [0.5, 0.6]\rangle$	$\langle[0.5, 0.6], [0.4, 0.5]\rangle$
a_4	$\langle[0.7, 0.8], [0.2, 0.3]\rangle$	$\langle[0.4, 0.5], [0.0, 0.1]\rangle$
a_5	$\langle[0.4, 0.5], [0.5, 0.6]\rangle$	$\langle[0.5, 0.6], [0.0, 0.1]\rangle$
	g_3	g_4
a_1	$\langle(0.5, 0.6, 0.7, 0.8), (0.2, 0.3, 0.4, 0.5)\rangle$	$\langle(0.4, 0.5, 0.6, 0.7), (0.0, 0.1, 0.2, 0.3)\rangle$
a_2	$\langle(0.4, 0.5, 0.6, 0.7), (0.0, 0.1, 0.2, 0.3)\rangle$	$\langle(0.5, 0.6, 0.7, 0.8), (0.6, 0.7, 0.8, 0.9)\rangle$
a_3	$\langle(0.2, 0.3, 0.3, 0.4), (0.3, 0.4, 0.4, 0.5)\rangle$	$\langle(0.1, 0.2, 0.3, 0.4), (0.5, 0.6, 0.7, 0.8)\rangle$
a_4	$\langle(0.3, 0.4, 0.5, 0.6), (0.4, 0.5, 0.6, 0.7)\rangle$	$\langle(0.4, 0.5, 0.6, 0.7), (0.1, 0.2, 0.3, 0.4)\rangle$
a_5	$\langle(0.6, 0.7, 0.8, 0.9), (0.2, 0.3, 0.4, 0.5)\rangle$	$\langle(0.3, 0.4, 0.5, 0.6), (0.5, 0.6, 0.7, 0.8)\rangle$

The DMs unanimously agreed to choose the exponential value function for the problem at hand, and their preferences for some parameters are $1 \leq p \leq 3$, $0.4 \leq \chi \leq 0.6$, $1 \leq \rho \leq 5$, $2 \leq \lambda \leq 2.5$.

At the initial stage of the analysis, the DMs provide no information on the weights of criteria. Case 1 is thus formulated based on the given information. Then we implemented the robustness analysis developed in Section 4.1. Tables 2-4 show the calculated results.

Table 2
The matrix of RAIs (in %) (Case 1)

	b_i^1	b_i^2	b_i^3	b_i^4	b_i^5
a_1	40.98	44.63	14.36	0.03	0
a_2	0.84	4.9	14.7	64.38	15.18
a_3	0	0	0	15.62	84.38
a_4	43.35	31.13	13.32	11.81	0.39
a_5	14.83	19.34	57.62	8.16	0.05

Table 3
The matrix of WPWIs (in %) (Case 1)

	a_1	a_2	a_3	a_4	a_5
a_1	50	98.28	100	50.53	77.75
a_2	1.72	50	84.82	13.24	12.06
a_3	0	15.18	50	0.39	0.05
a_4	49.47	86.76	99.61	50	69.4
a_5	22.25	87.94	99.95	30.6	50

Table 4
The matrix of CWVs and CCVs (Case 1)

	w_1	w_2	w_3	w_4	p	χ	ρ	λ
a_1	0.1748	0.1614	0.3454	0.3185	2.2204	3.0132	0.499	2.0114
a_2	0.0643	0.0743	0.7427	0.1187	2.2127	3.2209	0.5041	2.0114
a_3	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*
a_4	0.3712	0.2697	0.1162	0.2429	2.2239	2.9509	0.5011	1.9801
a_5	0.1178	0.4525	0.3443	0.0854	2.2189	3.1384	0.4983	1.9925

However, the DMs are not satisfied with the results in Case. They suggest to conduct a new round of analysis by providing additional preferences on criteria weights, that is, $w_1 \geq 2w_2, w_4 \geq w_3, 0.1 \leq w_1 \leq 0.4, w_2 \geq 0.05, 0.05 \leq w_3 \leq 0.15, w_4 \geq 0.05$, with the preferences for other parameters remain the same as in Case 1. Building upon this, Case 2 is thus formulated. Tables 5-6 shows some results.

Table 5
The matrix of RAIs (in %) (Case 2)

	b_i^1	b_i^2	b_i^3	b_i^4	b_i^5
a_1	34.12	65.88	0	0	0
a_2	0	0	15.41	83.8	0.79
a_3	0	0	0	0.86	99.14
a_4	65.88	34.12	0	0	0
a_5	0	0	84.59	15.34	0.07

Table 6
The matrix of WPWIs (in %) (Case 2)

	a_1	a_2	a_3	a_4	a_5
a_1	50	100	100	34.12	100
a_2	0	50	99.21	0	15.41
a_3	0	0.79	50	0	0.07
a_4	65.88	100	100	50	100
a_5	0	84.59	99.93	0	50

Based on Tables 5 and 6, we can derive a robust ranking of alternatives, that is, $a_4 \succ a_1 \succ a_5 \succ a_2 \succ a_3$. However, the DMs need to explain the results to stakeholders. Thus, we formulated the following model to determine representative criteria weights:

$$\vartheta^* = \max \vartheta$$
$$s.t. \begin{cases} T(i, E(\boldsymbol{x}), \boldsymbol{w}, E(\boldsymbol{s})) - T(h, E(\boldsymbol{x}), \boldsymbol{w}, E(\boldsymbol{s})) \geq \vartheta(p_{ih}^W - 0.5), \forall (i,h) \in M \times M : p_{ih}^W \geq 0.5, i \neq h \\ w_1 \geq 2w_2, w_4 \geq w_3 \\ 0.1 \leq w_1 \leq 0.4, 0.05 \leq w_3 \leq 0.15 \\ w_2 \geq 0.05, w_4 \geq 0.05 \\ \sum_{j=1}^4 w_j = 1 \end{cases} \tag{25}$$

By solving the model, we have $\vartheta^* = 2.4848$. Subsequently, by calculating the centroid of the sapce Ω^{ϑ^*}, we get the unique and most representative criteria weights $\boldsymbol{w} = (0.3435, 0.1253, 0.0500, 0.4812)$. Then we implemented the extended TODIM method, and get that $\bar{\pi}(a_1) = 0.8981$, $\bar{\pi}(a_2) = 0.2156$, $\bar{\pi}(a_3) = 0$, $\bar{\pi}(a_4) = 1$, and $\bar{\pi}(a_5) = 0.4214$. That is, the interpretable robust ranking of the alternatives under consideration is $a_4 \succ a_1 \succ a_5 \succ a_2 \succ a_3$.

5.2 *Comparative analysis*

To the best of our knowledge, two types of robustness analysis for TODIM methods have been developed in the literature, namely SMAA-TODIM [19] and SMAA-ExpTODIM [20]. However, neither of these methods is capable of handling generalized orthopair fuzzy information. Additionally, while both methods can derive a robust ranking, they lack the ability to provide an explanation for the resulting ranking. The method proposed in this paper addresses these gaps. More detailed analysis can be found in Table 7.

Table 7
Comparison analysis with other robust ranking methods

	The type of value function	Can the method handle generalized orthopair fuzzy information?	Is the ranking robust?	Is the ranking interpretable?
SMAA-TODIM [19]	Power value function	No	Yes	No
SMAA-ExpTODIM [20]	Exponential value function	No	Yes	No
The proposed approach	Power/exponential/logarithmic	Yes	Yes	Yes

6. Conclusions

This study introduces an interpretable robust TODIM approach within the context of generalized orthopair fuzzy settings for the first time. In the proposed approach, a grey relational coefficient-based relative closeness degree is introduced to extend the PowTODIM, ExpTODIM, and LogTODIM methods, adapting them to generalized orthopair fuzzy settings. Additionally, a set of robustness analysis measures is developed for the extended TODIM method, accounting for simultaneous uncertainty in criteria weights, value function coefficients, and the membership and non-membership degrees of generalized orthopair fuzzy sets. This enhancement enables the extended TODIM method to generate robust rankings in the presence of high-order uncertainty. Finally, a technique for determining representative criteria weights is developed to enhance the interpretability of the robust recommendations.

In this study, the criteria are assumed to be independent, an assumption that could be relaxed in future research to incorporate positive and negative interactions between them. In addition, extending the application of the proposed interpretable robust TODIM approach to recommendation systems with online reviews presents an intriguing opportunity, given the flexible representation of online reviews through generalized orthopair fuzzy sets.

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Conflicts of Interest

The authors declare no conflicts of interest.

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