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# On Some Aspects of Bounded Transportation Problem

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#### **ABSTRACT**

This paper presents various aspects of the capacitated transportation problem by incorporating into the classical transportation problem some realistic constraints such as limited capacities, restrictions on total flow, time-sensitive delivery of goods, and linear, quadratic, and fractional objectives. It delves into cost minimization, time minimization, and the trade-off between these two aspects. A special class of non-linear programming problems such as the fixed-charge bi-criterion transportation problem with an indefinite quadratic objective function with restriction on total flow is examined. The fractional problem, along with paradoxical scenarios where it is possible to ship more total goods for less total cost, is also presented. Each model is supported with mathematical formulations, some of which are illustrated through real-life applications such as the military transportation problem of the Indian Army.

#### 1. Introduction

Classical transportation problem is a class of linear programming problems where a homogenous commodity available at different warehouses is to be shipped to various markets and depots at minimum transportation cost/time. Pioneer work was done by Hitchcock [1] and Koopman [2] on these problems. Hitchcock formulated the transportation problem, and later on, Koopman construct a computationally more efficient solution approach that follows the steps of the simplex method. The most organized and effective technique for solving linear programming problems was simplex method. This technique was proposed by George Dantzig [3] in 1947. A necessary and sufficient condition for the existence of feasible solution of a transportation problem is that total demand is always equal to total supply. But in real world situations, there is a limited capacity of resources such as vehicles, docks, equipment etc. This gives rise to a capacitated transportation problem with bounds on rim conditions. Dahiya et al., [4] studied a class of capacitated transportation problem with bounds on rim conditions where only variable costs are taken in to account. Later, Adlakha et al., [5] studied a more realistic scenario of transportation problem in which both fixed and variable cost component is studied. Gupta [6] and Gupta and Arora [7,8] worked extensively on various aspects of bounded transportation problems but their work is centred on the methodology of

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classical optimization. In real life, the objective function is not always linear. Rather, the objective function is a product of two linear functions. Another class of non-linear programming problem is fractional programming problems where the objective function is the ratio of two linear functions. Optimization of a ratio of criteria often describes some kind of an efficiency measure for a system. Gupta and Arora [9] developed an algorithm to find optimum cost time trade off pairs in a fractional capacitated transportation problem with restricted flow. Dahiya and Verma [4] presented a note on two stage interval time minimization transportation problem. Some researchers such as Basu et al., [10] and Bhatia et al., [11] gave equal importance to both cost and time minimization. This dual objective give rise to optimum cost-time trade-off pairs in transportation problem. This paper is an attempt to summarize various aspects of capacitated transportation problem which finds its applications in real life. Many researchers have presented different techniques to solve capacitated transportation problems. Xie and Li [12] presented an iterative solution technique to minimize the average transportation cost of bounded transportation problem with bounds on rim conditions. Barma et al., [13] presented a genetic algorithm for the profit-maximizing capacitated vehicle routing problem under certain paradigm. This is further supported by the research of Jiang and Zhang [14] which demonstrates an improved adaptive differential evolution algorithm for the uncapacitated facility location problem. Pinacho-Davidson and Blum [15] presented a hybrid evolutionary algorithm for minimum bounded dominating set problem. An extension of this idea can be found in the work of Kulac and Kazancı [16] who used iterative optimization technique to optimize in-factory vehicle routing using swarm intelligence in plant logistics. However, Kaur et al., [17] offers a different viewpoint, suggesting an efficient algorithm for two-stage capacitated transportation problem based on classical optimization approach. Kumar and Dhanapal [18] used the technique of fermatean fuzzy numbers to solve the multi-objective bi-item capacitated transportation problem. This paper is an attempt to summarize various aspects of capacitated transportation problem which finds its applications in real life.

### 2. Defining Transportation problem

### 2.1 Classical Transportation Problem

A transportation problem may be described as follows: Let there be m sources with  $a_i$ ; i=1,2,3,...m units of supply of a particular commodity and n destinations having  $b_j$ ; j=1,2,3,...n units of demand respectively. It is assumed that  $a_i,b_j>0$ . Let  $c_{ij}$  be the unit cost of transportation from source  $i^{th}$  source to  $j^{th}$  destination. Since there is only one commodity, a destination can receive the demand from one or more sources. The problem is to determine the feasible shipping pattern from sources to destinations that minimizes the total transportation cost. The basic assumption here is that the transportation cost of a given route is directly proportional to the number of units transported. The definition of "unit of transportation" will vary depending on the commodity transported. Let  $x_{ij}$  be the number of units transported from source i to destination j. Assuming that

the total supply equals the total demand, that is  $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ , the mathematical formulation of the

standard transportation problem is: (P1): 
$$\min z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$
 subject to

$$\sum_{i,j} x_{ij} = a_i; i \in I$$

$$\sum_{i \in I} x_{ij} = b_j; j \in J$$

$$x_{ij} > 0; i \in I, j \in J$$

where  $I = \{1, 2, 3...m\}$  and  $J = \{1, 2, 3...n\}$ . However, in real life, it is not necessarily true that the total supply equals the total demand. In such situations, source and / or destinations constraints are inequations as opposed to the usual equations. Such unbalanced transportation problems can be studied by developing equivalent standard transportation problems.

### 2.2 Bounded Transportation Problem

The standard transportation problem, known as "Hitchcock – Koopmans Transportation Problem" is mathematically given by (P1). The total flow in the problem is  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ . But in real

world situations, there is a limited capacity of resources such as vehicles, docks, equipment etc. This gives rise to a capacitated transportation problem with bounds on rim conditions. A capacitated transportation problem is a distribution model with upper and lower bounds on the number of units shipped from an origin to a destination. This problem differs from the classical distribution model in which the node shipping amounts are, by contrast, specified exactly. This generalization of the classical distribution problem not only makes the model versatile from the theoretical stand point but also makes the model more usable from an application view point. In many applications of the distribution model, the firm is only interested in shipping exact number of units from each origin and in receiving an exact number of units at each destination

$$\min z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{1}$$

subject to

$$a_i \le \sum_{j \in J} x_{ij} \le A_i; i \in I \tag{2}$$

$$b_{j} \leq \sum_{i \in I} x_{ij} \leq B_{j}; j \in J \tag{3}$$

$$l_{ij} \le x_{ij} \le u_{ij}; i \in I, j \in J \text{ and integers.}$$
 (4)

where  $I = \{1, 2, ... m\}$  is the index set of m origins.  $J = \{1, 2, ..., n\}$  is the index set of n destinations.  $x_{ij}$  denotes the number of units to be transported from i<sup>th</sup> origin to j<sup>th</sup> destination.  $c_{ij}$  is the cost of transporting one unit of commodity from i<sup>th</sup> origin to j<sup>th</sup> destination.  $I_{ij}$  and  $I_{ij}$  are the bounds on number of units to be transported from i<sup>th</sup> origin to j<sup>th</sup> destination.  $I_{ij}$  and  $I_{ij}$  are the bounds on the availability at the i<sup>th</sup> origin,  $I_{ij} \in I_{ij}$  be and  $I_{ij} = I_{ij}$  and  $I_{ij} = I_{ij}$  be and  $I_{ij} = I_{ij}$  be a property of the destination.

#### 3. Aspects of Bounded Transportation Problem

### 3.1 Fixed Charge Capacitated Transportation Problem

Capacitated transportation problem finds its application in a variety of real-world problems like telecommunication networks, production – distribution system, rail and urban road system. But many distribution problems in practice can only be modelled as fixed charge transportation problems. For example – rails, roads and trucks have invariably freight rates which consists of fixed costs and variable costs both. The transportation cost in many distribution problems consists of fixed costs, which are independent of the amount transported and the variable costs, which are proportional to the amount shipped. For example – rail, road and truck companies invariably use

freight rates that comprise both fixed costs such as permit fees and property taxes and variable costs, such as direct equipment and personnel usage. Capacitated fixed charge transportation problem finds its application in warehouse location problem, in fleet routing, in scheduling problem etc. Capacitated transportation problem is solved by upper bounded simplex technique. The inclusion of upper bound in the transportation table requires modification in the feasibility condition of the simplex method because a basic variable can become a non-basic variable at its upper bound or lower bound. Moreover, when a non-basic variable becomes a basic variable, its value should not exceed its upper bound and should not be less than its lower bound. In addition to this, its value should not disturb the non-negativity and upper bound conditions of all existing basic variables. Mathematically,

a fixed charge capacitated transportation problem is represented by  $\min \left( \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i \right)$  subject

to the constraints (2), (3) and (4). Here,  $F_i$  represents the fixed cost associated with  $i^{th}$  origin. For the formulation of  $F_i$ ;  $i \in I$ , we assume that  $F_i$ ;  $i \in I$  has p number of steps so that  $F_i = \sum_{l=1}^p F_{il} \delta_{il}$ ;  $i \in I$  where

$$\delta_{il} = \begin{cases} 1; \sum_{j \in J} x_{ij} > a_{il} \\ 0; otherwise \end{cases} \forall l = 1, 2, 3 ... p; i \in I. \quad 0 = a_{i1} < a_{i2} < ... .< a_{ip} \; ; a_{i1}, a_{i2}, ..., a_{ip}; i \in I \; \text{are constants and} \; F_{il} \; \text{are}$$

the fixed costs for all  $i \in I; l = 1, 2, ...p$ .

### 3.2 Bottleneck Capacitated Transportation Problem

A special class of transportation problem called bottleneck transportation problem is considered where the objective is to minimize the maximum time of transporting all supply to the destinations under certain conditions. In a bottleneck transportation problem, the time of transporting items from origins to destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations. Time minimization is more important than cost minimization in certain situations such as when military units are to be sent from their bases to certain fronts within the shortest possible time. Another situation concerns the transportation of perishable goods. From practical point of view, the problem consists of supplying certain consumers with the necessary quantity of items (goods, military equipment, aircraft and so on) in such a way that the total time from the starting of the operation until its completion should be minimal. This section studies the objective function of time minimization in a capacitated transportation problem when decision variables are bounded. A technique for minimizing time in a capacitated transportation problem with bounds on rim conditions is developed. The procedure involves finite iterations and is based on movement from one extreme point to another extreme point until an optimal solution is reached. The technique is based on certain assumptions such as the carriers have sufficient capacity to carry goods from an origin to a destination in a single trip. Moreover, they start simultaneously from their respective origins. The mathematical model of the problem is:  $\min T = \max\{t_{ij} \mid x_{ij} > 0\}$  subject to the constraints (2), (3) and (4). Here,  $t_{ij}$  is the time of transporting goods from  $i^{th}$  origin to the  $j^{th}$ destination. For any given feasible solution,  $X = x_{ii}$  satisfying (2), (3) and (4), the time of transportation is the maximum of t<sub>ij</sub>,s among the cells in which there are positive allocations i.e. corresponding to the solution X, the time of transportation is  $\left\{\max_{(i,j)} \left(t_{ij} \mid x_{ij} > 0\right)\right\}$ . This problem can be used to solve the real-life military transportation problem of Indian Army discussed in next section.

### 3.3 Military Transportation Problem of Indian Army

Army Headquarters at Pathankot supply military units with arms, ammunitions, food etc. to various locations at J and K border from its two regiments (i) Dogra and Sikh. Indian Army used to supply the military units to three crucial locations (j) –Kargil, Poonch and Rajouri of J.K border. Let  $x_{ij}$  be the quantity of military units with arms, ammunitions, food etc. sent from the  $i^{th}$  regiment to the  $j^{th}$  location. Let  $t_{ij}$  be the time of transporting the military units from the  $i^{th}$  regiment to the  $j^{th}$  location. Then the goal is to determine the transportation schedule which minimizes the maximum time of transporting the military units with arms, ammunitions and food etc., during emergency situations. The problem can be formulated as follows:

$$\begin{aligned} &\min \, \mathsf{z} = \max_{(i,j)} \{t_{ij} \mid x_{ij} > 0\} \\ &\mathsf{subject to} \\ &\mathsf{5} \leq \sum_{j=1}^3 x_{1j} \leq 30, \quad 10 \leq \sum_{j=1}^3 x_{2j} \leq 40 \;, \; 10 \leq \sum_{i=1}^2 x_{i1} \leq 30, \; \; 5 \leq \sum_{i=1}^2 x_{i2} \leq 20 \;, \; 5 \leq \sum_{i=1}^2 x_{i3} \leq 30 \\ &1 \leq x_{11} \leq 10; 2 \leq x_{12} \leq 10; 0 \leq x_{13} \leq 5 \\ &0 \leq x_{21} \leq 15; 1 \leq x_{22} \leq 10; 1 \leq x_{23} \leq 20 \\ &t_{11} = 15; t_{12} = 8; t_{13} = 2; t_{21} = 16; t_{22} = 14; t_{23} = 11 \end{aligned}$$

On applying the algorithm for solving the bounded transportation problem described by Gupta and Arora [7], the decision maker will get the minimum time of transporting goods is 15 and the amount in pipeline is 6 units.

### 3.4 Cost-time Trade-off in a Bounded Transportation Problem

The fixed charge transportation problem can be stated as a distribution problem in which there are m suppliers (warehouses or factories) and n customers (destinations or demand points). Each of the m supplies can ship to any of the n customers at a shipping cost per unit  $c_{ii}$  (unit cost for shipping from supplier i to customer j) plus a fixed cost Fi. Sometimes, the total capacity of each route is also specified by some external decision maker because of budget or other political consideration. This gives rise to capacitated time minimizing transportation problems. Moreover, sometimes a fixed cost (like set up cost for machines, landing fees at an airport, cost of renting a vehicle) is also associated with every origin due to which fixed charge must also be taken in to account along with variable cost of transporting goods from various origins to different destinations. Normally, bi – criterion problem arises when the user has to compromise between two criteria. In emergency situations such as fire services, ambulance services, police services etc., when the time of transportation is more important than cost of transportation. If the total flow in a transportation problem with bounds on rim conditions is also specified, the resulting problem makes the transportation problem more realistic. Moreover, if the total capacity of each route is also specified then optimal solution of such problems is of greater importance which gives rise to capacitated transportation problems. From the practical point of view, the cost minimizing transportation problem and the time minimizing transportation problem cannot be viewed as two independent problems. The mathematical model of the problem in which the two objectives of minimizing cost and time are unified is given below.

$$\min \left\{ \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i, \max_{i \in I, j \in J} \left( t_{ij} \mid x_{ij} > 0 \right) \right\}$$
 subject to (2), (3) and (4). These types of problems have

practical importance in real life.

#### 3.5 Indefinite Quadratic Capacitated Transportation Problem with Restricted Flow

In linear programming, the values of decision variables are to be determined so as to optimize the value of the linear objective function subject to linear constraints. However, when either objective function or constraints or both are not expressed in terms of linear relationships among decision variables, we take the help of non -linear programming methods to solve such non-linear programming problems. A special class of transportation problems called indefinite quadratic transportation problem consist of the objective function as the product of two linear functions. One of the linear- function represents the total variable cost of transporting goods from various supply points to various demand points. Another linear function represents the total damage cost or depreciation cost which is incurred while transporting goods. Practically, a fixed cost called set up cost is also incurred when a commodity is transported. The objective function discussed in this section considers fixed charge also. In addition to cost, a time is also associated with each shipping route. A business man must be interested in minimizing the maximum time of transporting all supply to the destinations. From the practical point of view, the cost minimizing transportation problem and the time minimizing transportation problem cannot be viewed as two independent problems. This gives rise to efficient time cost trade off pairs which minimizes cost and time simultaneously in a capacitated fixed charge bi-criterion indefinite quadratic transportation problem. The mathematical model of such a problem as:

$$\min\left\{\left(\sum_{i\in I}\sum_{j\in J}c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J}d_{ij}x_{ij}\right)+\sum_{i\in I}F_{i},\max_{i\in I,j\in J}\left(t_{ij}\mid x_{ij}>0\right)\right\}\text{ subject to (2),(3) and (4)}.$$

In any transportation problem, the total quantity supplied by the various supply points and consequently received by the various destinations is the total flow in the system. This flow is different for different combinations of supply points and destination constraints. Sometimes, situations arise where either reserve stocks have to be kept at the supply points say, for emergencies or there is a shortfall in the production level. In such situations, the total flow needs to be curtailed. This gives rise to a new restricted flow constraint given by  $\sum_{i \in I} \sum_{j \in J} x_{ij} = P \text{ in the transportation problem where}$ 

$$P < \min \left( \sum_{i \in I} a_i, \sum_{j \in J} b_j \right)$$
. This gives rise to a capacitated fixed charge bi – criterion indefinite quadratic

transportation problem with restriction on total flow. This type of problems can be used in solving the problems of industry as discussed by Gupta and Arora [7,8].

#### 3.6 Fractional Transportation Problem

There is a wide scope of fractional transportation problem [19-21] in practice such as stock cutting problem, resource allocation problems, routing problems for ships and planes, cargo loading problem, inventory problem and many others. The standard transportation problem aims at minimizing the total cost of transporting a uniform product from various supply points to various destinations. But the objective function of profit maximization is not considered so far. We now consider an objective function which minimizes the total variable cost of transporting goods from various sources to different destinations and simultaneously maximizes the total variable profit earned when goods are transported. In addition to this, the objective function maximizes the rate of return on a fixed capital investment. The mathematical model of such a problem is given by

$$\min \left\{ \frac{\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij}} + \frac{\sum_{i \in I} F_i}{\sum_{i \in I} G_i} \right\}$$

subject to (2), (3), (4) and (5).

#### 3.7 Paradox in a Bounded Transportation Problem

A paradoxical situation arises when value of the objective function falls below the optimal value and this lower value is attainable by transporting larger amount of quantity. The source of the so-called transportation paradox is unclear. Apparently, many researchers have discovered independently from each other the following behavior of the transportation problem. In certain cases of the transportation problem, an increase in the supplies and demand may lead to a decrease in the optimal transportation cost. In other words, by moving bigger amount of goods around, one may save a lot of money. This surely sounds paradoxical. The more for less paradox in a transportation problem occurs when it is possible to ship more total goods for less (or equal) total cost, while shipping the same amount or more from each origin to each destination and keeping all shipping costs non negative. The information of occurrence of a paradoxical situation is useful to a manager in deciding which warehouse or plant capacities are to be increased and which market should be sought. It could also be a useful tool in analyzing and planning company acquisitions, mergers, consolidations and downsizes. If a paradox exists, one would obviously be interested in the best paradoxical pair.

#### 4. Model Comparison

The classical and capacitated transportation problems prioritize minimizing transportation costs. These models are more suitable for logistics problems where costs are the primary concern. The bottleneck and time-minimization models prioritize rapid delivery, vital in emergency services or perishable goods supply chains. The fixed charge, cost-time trade-off, indefinite quadratic, and fractional transportation models incorporate multiple objectives, adding complexity but offering more realistic solutions where multiple factors (cost, time, profit) influence decision-making. Quadratic and fractional models address non-linear relationships like depreciation costs and profit-to-cost ratios, which are critical while studying resource allocation and industrial planning. Each model has its strengths and limitations, depending on the specific constraints and objectives of the transportation scenario being addressed. These models offer varying levels of complexity and practical utility.

#### 5. Conclusion

This paper provides a comprehensive overview of the capacitated transportation problem including linear and non-linear objectives. It highlights the importance of integrating constraints such as rim conditions, restricted flow, and specified flow in transportation planning. The mathematical formulations of these problems and their applications in real-world scenarios, such as telecommunication networks, warehouse management, and emergency logistics to warehousing, were discussed. Techniques for finding optimal cost-time trade-off pairs were also examined, particularly in cases where minimizing transportation time is critical, such as in military operations and perishable goods transportation. Overall, this paper provides a comprehensive summary of the capacitated transportation problem, with an emphasis on its practical applications in logistics and resource management. Despite the extensive analysis presented, there are certain limitations. First, the mathematical models developed in this paper are based on simplifying assumptions, such as simultaneous dispatch of carriers and sufficient carrier capacity. These assumptions might not always hold in real-world transportation systems. Moreover, the study does not account for dynamic or stochastic variables, such as uncertain demand or transportation delays, which could significantly affect the outcome of transportation planning. As future work is intended, we can extend the models

to accommodate multi-modal transportation systems or integrating them with modern technologies such as blockchain or Artificial intelligence for real-time optimization. This could further improve the practical applicability of these methods in diverse sectors.

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#### **Conflicts of Interest**

The authors declare no conflicts of interest.

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