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Sustainable Urban Development: q-Rung Orthopair Fuzzy MCDM with Generalized Power Prioritized Yager Aggregation

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ABSTRACT

In light of accelerating urbanization and the intensifying challenges of climate change, ensuring sustainability and resilience in urban environments has become a strategic imperative. This study introduces an innovative methodology that combines Multi-Criteria Decision Making (MCDM) with q-rung orthopair fuzzy Yager aggregation operators (q-ROFYAOs) to address the multifaceted complexities of sustainable urban development. The paper proposes novel Yager operations grounded in Yager t-norms within the framework of q-rung orthopair fuzzy sets (q-ROFSs). Utilizing these foundations, two advanced aggregation operators are formulated: the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average Operator (q-ROFGPOPRYWA), and the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Geometric Operator (q-ROFGPOPRYWG). These operators satisfy essential mathematical properties, including idempotency, monotonicity, and boundedness, ensuring consistency and reliability in decision-making contexts. To demonstrate the practical relevance of the proposed approach, the operators are embedded within an MCDM framework and applied to a real-world case study on sustainable urban planning. The analysis encompasses sensitivity testing, comparative evaluations, and performance assessments, offering comprehensive insights into the robustness of the method. Finally, the paper provides a critical evaluation of the advantages and limitations of the proposed operators, underscoring their effectiveness in promoting urban resilience and minimizing environmental impact within complex decision-making environments.

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1. Introduction

Addressing contemporary urban challenges requires innovative and adaptive strategies. Advancements in intelligent technologies play a pivotal role in promoting sustainable urban development by enhancing resource efficiency, optimizing infrastructure, and strengthening urban resilience. This interdisciplinary approach supports smarter city planning and data-driven decision-making, enabling cities to respond dynamically to environmental and social pressures. Integrating such technologies into urban systems is essential for fostering innovation, resilience, and long-term sustainability. For the full forms of abbreviations used throughout this paper, please refer to the table provided in the Appendix.

1.1 *Motivation for the Proposed Research*

Urban planning is currently undergoing a significant transformation driven by the integration of advanced technologies. These innovations have the potential to reshape and redefine the urban planning landscape. However, numerous questions remain regarding their broader impacts on both theoretical research and practical implementation in urban and regional contexts, including the challenges they pose and the strategies required to address them effectively [1]. As cities strive to enhance sustainability, there is an increasing need for innovative, technology-enabled solutions. Allam et al. [2] emphasize the importance of critically engaging with new technologies and advocate for their thoughtful integration into the societal framework. To this end, careful calibration and contextual adaptation are essential for building cities that are not only intelligent but also genuinely sustainable and resilient. Advancements in urban systems can support resource optimization, improve infrastructure performance, and drive innovation. Achieving sustainable urban development ultimately requires a comprehensive approach that balances environmental, economic, and social dimensions.
The Multi-Criteria Decision-Making (MCDM) method stands as a potent approach in decision-making, providing a thorough assessment of multiple criteria, addressing subjectivity, ensuring transparency, and accommodating both quantitative and qualitative data [3–6]. MCDM represents a contemporary approach aimed at identifying the most favorable alternative that maximizes profit in accordance with attribute values. The theory and methodologies of MCDM find application in making significant decisions across various domains, including personnel selection, industrialization, waste management, site selection, urban innovation, and more. The MCDM procedure involves three crucial steps. Firstly, information about alternatives is gathered based on diverse attributes. Subsequently, the collected information is aggregated to determine the overall decision value for the target. The final step involves selecting the best option after ranking the alternatives in order of preference [7].
q-rung orthopair fuzzy set (q-ROFS) is an extension of orthopair fuzzy sets that offers greater granularity for representing uncertainty and preferences. This enhanced expressiveness makes q-ROFS especially valuable in fields such as urban planning, environmental management, and decision-making under uncertainty. Building on prior research in sustainable energy planning [8], our study employs q-ROFS and Yager operators to support sustainable urban innovation and resilience by effectively addressing uncertainties in urban decision-making.

1.2 *Literature Review*

Traditional approaches in formal computing often produce precise, binary outcomes—typically expressed as a definitive yes or no. However, this binary logic fails to capture the subtleties inherent in many real-world scenarios. To overcome this limitation, Zadeh introduced the Fuzzy Set (FS) theory in 1965 [9], which allows elements to possess membership grades within the unit interval [0, 1]. Despite this advancement, FSs rely solely on membership values (MV), which can be insufficient for complex decision-making contexts. Recognizing the need for richer information, Atanassov [10] introduced Intuitionistic Fuzzy Sets (IFSs), which incorporate both membership and non-membership values (NMV). IFS theory has since been widely adopted in multiple criteria decision-making (MCDM). For example, Seikh and Mandal [11] introduced Dombi aggregation operators (AOs) for integrating job data in an intuitionistic fuzzy environment, while Senapati et al. [12] proposed Aczel-Alsina operators for sustainable transportation practices. Other researchers, such as Gohain et al. [13], applied symmetric distance measures for pattern recognition and clustering, and Ke et al. [14] devised a ranking method for site selection in photovoltaic poverty alleviation projects. Wan and Yi [15] extended this work with power average operators for trapezoidal IFSs using strict t-norms and t-conorms. A key limitation of IFSs is that the sum of MV and NMV must not exceed 1, which restricts their applicability in more complex uncertainty modeling. To address this, Yager [16] introduced Pythagorean Fuzzy Sets (PyFS), where the sum of the squares of MV and NMV is restricted to 1. PyFS has proven effective for addressing complex MCDM problems [17, 18]. Building on this, Yager later introduced the q-rung orthopair fuzzy set (q-ROFS) [19], where the q-th powers of MV and NMV must sum to ≤ 1, allowing even greater flexibility in handling uncertainty.

Numerous applications of q-ROFSs have emerged: Wang et al. [20] proposed a q-ROF MABAC model for MADM problems; Seikh and Mandal [7] developed q-ROF Frank AOs; Wang et al. [21, 22] introduced Muirhead mean and Hamy mean operators for decision fusion and ERP systems, respectively; and Kausar et al. [23] adapted the CODAS method for q-ROFSs in cancer risk assessment. Aggregation operators are essential for simplifying complex datasets, offering functions like sum, average, and count to extract meaningful insights. They are especially valuable in high-dimensional decision environments where interpretability and precision are critical. In our study, we explore the role of artificial intelligence in urban planning and propose a new class of q-rung orthopair fuzzy Yager aggregation operators (q-ROFYAO). We thoroughly examine their mathematical properties and demonstrate their ability to model uncertainty and nonlinear relationships with high precision. These operators, grounded in Yager's t-norm, are particularly effective for capturing complex interdependencies, making them well-suited to applications in control systems, optimization, and decision-making processes. To validate our approach, we apply the q-ROFYAO within MCDM framework aimed at promoting sustainable urban innovation and resilience.

1.3 Research Gap

Before presenting the main contributions of this study, it is important to highlight the research gaps that motivated our work. These include the limited exploration of ExpoLogarithmic t-norms and t-conorms within the q-rung orthopair fuzzy set (q-ROFS) framework, and the lack of a solid theoretical foundation for Yager aggregation tools. Moreover, the absence of specialized aggregation operators for q-ROFSs and the insufficient validation of such operators in practical MCDM applications further underscore the need for this research. Addressing these gaps forms the core of our study's contributions.

1.4 Research Questions

To guide this study, the following research questions are proposed:

(i). What are the key properties and contributions of Yager t-norms and t-conorms in the context of fuzzy set theory?

(ii). How do Yager aggregation tools operate within the q-rung orthopair fuzzy set (q-ROFS) framework, and how do they support smooth and effective aggregation?

(iii). In what ways do the proposed operators—q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average (q-ROFGPOPRYWA) and Weighted Geometric (q-ROFGPOPRYWG) Operators—differ from traditional methods, and what are their essential properties with illustrative examples?

(iv). How can the proposed operators be validated through a real-world MCDM problem focused on sustainable urban innovation and resilience using artificial intelligence?

1.5 *Issues with Earlier Works*

The proposed study is motivated by several gaps in the existing literature, including:

(i). Limited exploration and application of Yager t-norms and t-conorms in fuzzy set theory.

(ii). Inadequate analysis of Yager aggregation operations within the q-ROFS context.

(iii). Lack of specialized aggregation operators such as q-ROFGPOPRYWA and q-ROFGPOPRYWG in existing research.

(iv). Limited empirical validation of proposed fuzzy operators in real-world MCDM applications, especially within the context of sustainable urban development.

1.6 *Main Contributions of the Study*

To address the identified research gaps, this study makes the following key contributions:

(i). Introduction of Yager t-norms and t-conorms: The study introduces the Yager t norms and t norms and explores their mathematical properties.

(ii). Formulation of Fundamental Operations: It details the core operations of Yager aggregation tools within the fuzzy set of orthopair q-rung (q-ROFS) framework, offering a foundation for smooth and effective aggregation.

(iii). Development of Novel Aggregation Operators: Two new operators are proposed: q-Rung orthopair fuzzy generalized power prioritized yager weighted average (q-ROFGPOPRYWA) and weighted geometric (q-ROFGPOPRYWG)—with an analysis of their properties and illustrative examples.

(iv). Validation via Real-World Application: The practical utility of the proposed operators is demonstrated through a real-world MCDM case study centered on sustainable urban development.

1.7 *Organization of the Proposed Study*

The structure of this study is organized as follows: Section 2 presents the fundamental concepts that form the theoretical foundation of the research. Section 3 introduces Yager's operations on q-rung orthopair fuzzy numbers, highlighting their formulation and significance. Section 4 proposes the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Aggregation Operators and examines their mathematical properties in detail. Section 5 outlines the Multi-Criteria Decision-Making (MCDM) methodology, accompanied by a sensitivity analysis and a discussion of the advantages and limitations of the proposed operators. Section 6 concludes the study by summarizing the key findings and implications. Finally, Section 7 provides an appendix with a tabular list of abbreviations used throughout the paper.

2. Methodology

2.1 Background and Fundamentals

This section outlines the foundational concepts in fuzzy set theory, forming the basis for the analyses presented in the later sections. Definitions 2.1, 2.2, 2.3, and 2.4 introduce key constructs, including fuzzy sets (FS), intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets (PyFS), and q-rung orthopair fuzzy sets (q-ROFS). Example 2.1 illustrates the practical relevance of q-rung orthopair fuzzy numbers (q-ROFNs) through a real-world scenario. The section also explores Yager's t-norm and t-conorm, followed by discussions on Yager's Power Average (PA) and Prioritized Average (PRA) operators, which are integral to the aggregation techniques developed later in the study. Unless otherwise specified, let $\mathcal{X}$ denote a non-empty universal set throughout this paper. Definition 2.5 introduces the score function for q-ROFNs, while Definition 2.6 explains how to compare these score values. Overall, this section establishes both the theoretical underpinnings and the practical context for applying q-ROFS theory, providing a solid foundation for the advanced methods and applications discussed in subsequent sections. Here, we define a Fuzzy Set $\mathcal{F}$ over $\mathcal{X}$ as follows:

Definition 2.1. *[9] The concept of a fuzzy set (FS) is defined as follows:*

$$\mathcal{F} = \{(x, \alpha(x)) : x \in \mathcal{X}\}$$

Here, $\alpha(x)$ is the membership grade of an element $x \in \mathcal{X}$, while $\alpha(x)$ is restricted to values within the interval $[0, 1]$.

Clearing the path for the examination of uncertainty, Atanassov [10] introduced the concept of intuitionistic fuzzy set (IFS) as an innovative framework designed to capture the nuanced aspects inherent in decision-making processes.

Definition 2.2. *[10] The concept of an intuitionistic fuzzy set (IFS) is defined as follows:*

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathcal{X}\}$$

Here, $\alpha(x), \beta(x)$ are the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x) + \beta(x) \leq 1$.

Ronald R. Yager, as outlined in his publication [16], introduced the Pythagorean fuzzy set (PyFS) framework, offering a novel approach to encapsulate uncertainty within the domain $\mathcal{X}$. The distinctive representation is formulated as follows:

Definition 2.3. *[16] The concept of a Pythagorean fuzzy set (PyFS) is defined as follows:*

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathcal{X}\}$$

Here, $\alpha(x), \beta(x)$ are the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x)^2 + \beta(x)^2 \leq 1$.

In his pioneering research [19], Ronald R.Yager introduced the revolutionary concept of q-rung orthopair fuzzy set (q-ROFS) $\mathcal{F}$ over $\mathcal{X}$, offering a unique characterization that extends beyond the traditional fuzzy set framework.

Definition 2.4. *[19] The concept of a q-rung orthopair fuzzy set (q-ROFS) is defined as follows:*

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathcal{X}\}$$

Here, $\alpha(x)$ and $\beta(x)$ represent the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x)^q + \beta(x)^q \leq 1$ $(q \geq 1)$. Furthermore, a q-rung orthopair fuzzy number (q-ROFN) is symbolized as $\mathcal{F} = (\alpha, \beta)$ or $\mathcal{F}_t = (\alpha_t, \beta_t)$ for convenience (where t is a positive integer).

Liu et al. [24] proposed a score function for any q-ROFN, represented as $\mathcal{F} = (\alpha, \beta)$, defined by $\mathcal{S}(\mathcal{F}) = \alpha^q - \beta^q$. The resulting value of $\mathcal{S}(\mathcal{F})$ may initially fall within the interval $[-1, 1]$. To enhance computational convenience and ensure that $\mathcal{S}(\mathcal{F})$ resides within the more practical interval $[0, 1]$, we have made a slight modification to Liu et al. [24]'s score function for q-ROFN as follows.

Definition 2.5. *The score function for any q-rung orthopair fuzzy number (q-ROFN), $\mathcal{F} = (\alpha, \beta)$, is defined as follows:*

$$\mathcal{S}(\mathcal{F}) = 1/2 + (1/2)(\alpha^q - \beta^q) \tag{1}$$

Liu et al. [24] revolutionize fuzzy set theory by presenting a novel perspective that sheds light on intricate relationships within q-rung orthopair fuzzy sets.

Definition 2.6. *[25] For any two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$, the relationship between the score values of $\mathcal{F}_1$ and $\mathcal{F}_2$ can be expressed as:*

$$\text{If } \mathcal{S}(\mathcal{F}_1) < \mathcal{S}(\mathcal{F}_2) \text{ then } \mathcal{F}_1 < \mathcal{F}_2$$

$$\text{If } \mathcal{S}(\mathcal{F}_1) > \mathcal{S}(\mathcal{F}_2) \text{ then } \mathcal{F}_1 > \mathcal{F}_2$$

$$\text{If } \mathcal{S}(\mathcal{F}_1) = \mathcal{S}(\mathcal{F}_1) \text{ then } \mathcal{F}_1 = \mathcal{F}_2$$

Drawing inspiration from Yager's pioneering contributions in fuzzy set theory [19], this definition introduces Yager's t-norm (YTN) and t-conorm (YTCN), thereby making a significant contribution to the fundamental principles of fuzzy logic.

Definition 2.7. *[19] For any two $x, y \in [0, 1]$, and $\mu \in (0, \infty)$, Yager's t-norm (YTN) and Yager's t-conorm (YTCN) are defined as follows.*

$$\text{YTN}(x, y) = \min\{1, (x^\mu + y^\mu)^{1/\mu}\}$$

$$\text{YTCN}(x, y) = 1 - \min\{1, ((1 - x)^\mu + (1 - y)^\mu)^{1/\mu}\}$$

Definition 2.8. *[26] Consider a collection of criteria $\mathcal{F}_i$ for $i = 1, 2, \ldots, m$. The Power Average (PA) Operator is defined as:*

$$\text{PA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i, \tag{2}$$

where the Power coefficients η_i are computed as:

$$\eta_i = \frac{1 + \mathcal{U}_i}{\sum_{k=1}^{m}(1 + \mathcal{U}_k)},$$

and $\mathcal{U}_i = \sum_{\substack{k=1 \\ k \neq i}}^{m} [1 - |\mathcal{F}_i - \mathcal{F}_k|]$ for $i = 1, 2, \ldots, m$.

Definition 2.9. *[27] Consider a collection of criteria $\mathcal{F}_i$ for $i = 1, 2, \ldots, m$. The Prioritized Average (PRA) Operator is defined as:*

$$\text{PRA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i,$$

where the Prioritized coefficients η_i are computed as:

$$\eta_i = \frac{\mathcal{V}_i}{\sum_{k=1}^{m} \mathcal{V}_k},$$

with $\mathcal{V}_1 = 1$ and $\mathcal{V}_i = \sum_{k=1}^{i-1} \mathcal{S}(\mathcal{F}_i)$ $(i = 2, \ldots, m)$.

3. Yager's Operations on q-Rung orthopair Fuzzy Numbers

In this section, we introduce Yager's operations, which include $\mathcal{F}_1 \oplus \mathcal{F}_2$, $\mathcal{F}_1 \otimes \mathcal{F}_2$, $\delta\mathcal{F}_1$, and $\mathcal{F}_1^{\delta}$ $(\delta > 0)$, applied specifically to q-rung orthopair fuzzy numbers. These operations leverage Yager's t-norm and t-conorm, as discussed in Definition 2.7.

To illustrate the practical application of these Yager's operations on q-rung orthopair fuzzy numbers, we examine a real-life scenario: the sustainable urban development for Green Infrastructure Project within urban areas. Additionally, we discuss the essential properties of these operations, although formal proofs are not provided.

Definition 3.1. *[16] The operations $\oplus$ and $\otimes$ by Yager between two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$, with $\mu, \delta > 0$, are defined as follows:*

$$\begin{aligned}
\mathcal{F}_1 \oplus \mathcal{F}_2 &= \left(\left[1 - \min\{1, ((1-\alpha_1^q)^{\mu} + (1-\alpha_2^q)^{\mu})^{1/\mu}\}\right]^{1/q}, \left[\min\{(1, ((\beta_1^q)^{\mu} + (\beta_2^q)^{\mu})^{1/\mu}\}\right]^{1/q} \right) \\
\mathcal{F}_1 \otimes \mathcal{F}_2 &= \left(\left[\min\{1, ((\alpha_1^q)^{\mu} + (\alpha_2^q)^{\mu})^{1/\mu}\}\right]^{1/q}, \left[1 - \min\{1, ((1-\beta_1^q)^{\mu} + (1-\beta_2^q)^{\mu})^{1/\mu}\}\right]^{1/q} \right) \\
\delta\mathcal{F}_1 &= \left(\left[1 - \min\{1, (\delta(1-\alpha_1^q)^{\mu})^{1/\mu}\}\right]^{1/q}, \left[\min\{1, (\delta(\beta_1^q)^{\mu})^{1/\mu}\}\right]^{1/q} \right) \\
\mathcal{F}_1^{\delta} &= \left(\left[\min\{1, (\delta(\alpha_1^q)^{\mu})^{1/\mu}\}\right]^{1/q}, \left[1 - \min\{1, (\delta(1-\beta_1^q)^{\mu})^{1/\mu}\}\right]^{1/q} \right)
\end{aligned} \tag{3}$$

To illustrate the operations by Yager on q-rung orthopair fuzzy numbers, represented as $\mathcal{F}_1 \oplus \mathcal{F}_2$, $\mathcal{F}_1 \otimes \mathcal{F}_2$, $\delta\mathcal{F}_1$, and $\mathcal{F}_1^{\delta}$, as defined in Definition 3.1, we delve into the practical application of the sustainable urban development for Green Infrastructure Project within urban areas as outlined in Example 3.2.

Example 3.2. *In the evaluation of sustainable urban development initiatives, such as "Green Infrastructure Project Y", professionals focus on specific criteria to ensure environmental resilience and community well-being. This project aims to integrate green spaces and sustainable infrastructure into urban areas, thereby enhancing quality of life and mitigating environmental risks.The evaluation of "Green Infrastructure Project Y" includes the following criteria:*

(i). Green Space Accessibility (Criteria 1): Access to green spaces plays a crucial role in promoting urban residents well-being and environmental sustainability [28]. By ensuring easy access to parks, gardens, and recreational areas, cities can enhance physical and mental health while fostering a deeper connection with nature.

(ii). Stormwater Management (Criteria 2): Effective management of stormwater is vital for reducing flood risks and preserving water quality in urban areas [29, 30]. Implementing sustainable drainage systems and green roofs can help mitigate the impact of heavy rainfall events, contributing to overall resilience against climate change.

To quantify these criteria comprehensively, intelligent systems technology and q-rung orthopair fuzzy numbers are employed.

For Green Space Accessibility (Criteria 1), represented by $\mathcal{F}_1 = (0.7, 0.6)$, a membership grade of 0.7 indicates high accessibility to green spaces, while a non-membership grade of 0.6 denotes lower accessibility.

Similarly, for Stormwater Management (Criteria 2), represented by $\mathcal{F}_2 = (0.8, 0.7)$, a membership grade of 0.8 indicates effective stormwater management practices, while a non-membership grade of 0.7 suggests areas for improvement.

By applying Yager's operations on q-rung orthopair fuzzy numbers (such as $\oplus$ and $\otimes$), urban planners can integrate these criteria to assess the overall effectiveness of "Green Infrastructure Project Y"in promoting sustainable urban development and resilience. While this example simplifies the evaluation to two criteria, real-world assessments of sustainable urban projects would consider a broader range of factors, including biodiversity conservation, energy efficiency, and social equity, to ensure comprehensive and effective urban planning and development strategies.

To assess the effectiveness of sustainability Regimen Y, Yager's operations are utilized on q-rung orthopair fuzzy numbers ($q = 4$), represented as $\mathcal{F}_1 \oplus \mathcal{F}_2$ and $\mathcal{F}_1 \otimes \mathcal{F}_2$, with the parameter μ fixed at 2 according to Definition 3.1. The process for computing $\mathcal{F}_1 \oplus \mathcal{F}_2$ is delineated in Equation (4).

$$\mathcal{F}_1 \oplus \mathcal{F}_2 = (\alpha, \beta) \tag{4}$$

Here, α is calculated using the provided data and Equation (3.1):

$$\begin{aligned}
\alpha &= \left[1 - \min\left(1, ((1-\alpha_1^q)^\mu + (1-\alpha_2^q)^\mu)^{1/\mu}\right)\right]^{1/q} \\
&= \left[1 - \min\left(1, \left((1-0.7^4)^2 + (1-0.8^4)^2\right)^{1/2}\right)\right]^{1/4} \\
&= [1 - \min(1, 0.0.962299)]^{1/4} \\
&= 0.4406431
\end{aligned}$$

Similarly, the value of β in Equation (4) is determined using Equation (3.1) along with the provided data. The resulting Yager's operation, denoted as $\mathcal{F}_1 \oplus \mathcal{F}_2$, is expressed as:

$$\mathcal{F}_1 \oplus \mathcal{F}_2 = (0.4406431, 0.7227346)$$

Likewise, the computation of $\mathcal{F}_1 \otimes \mathcal{F}_2$ yields:

$$\mathcal{F}_1 \otimes \mathcal{F}_2 = (0.8300879, 0)$$

Applying the score function defined in Equation (1), we obtain the effectiveness scores: $\mathcal{S}(\mathcal{F}_1 \oplus \mathcal{F}_2) = 0.3824280$ and $\mathcal{S}(\mathcal{F}_1 \otimes \mathcal{F}_2) = 0.5791307$. Thus, the effectiveness of sustainability Regimen Y, as determined by Yager's operations on q-rung orthopair Fuzzy Numbers ($q = 4$), is 0.3824280 for $\mathcal{F}_1 \oplus \mathcal{F}_2$ and 0.5791307 for $\mathcal{F}_1 \otimes \mathcal{F}_2$. The values of the remaining Yager's operations, $\delta\mathcal{F}_1$ and $\mathcal{F}_1^\delta$, can be similarly determined for a fixed value of δ. Figure 1 enhances the visualization of sustainable urban development for the Green Infrastructure Project as discussed in Example 3.2.

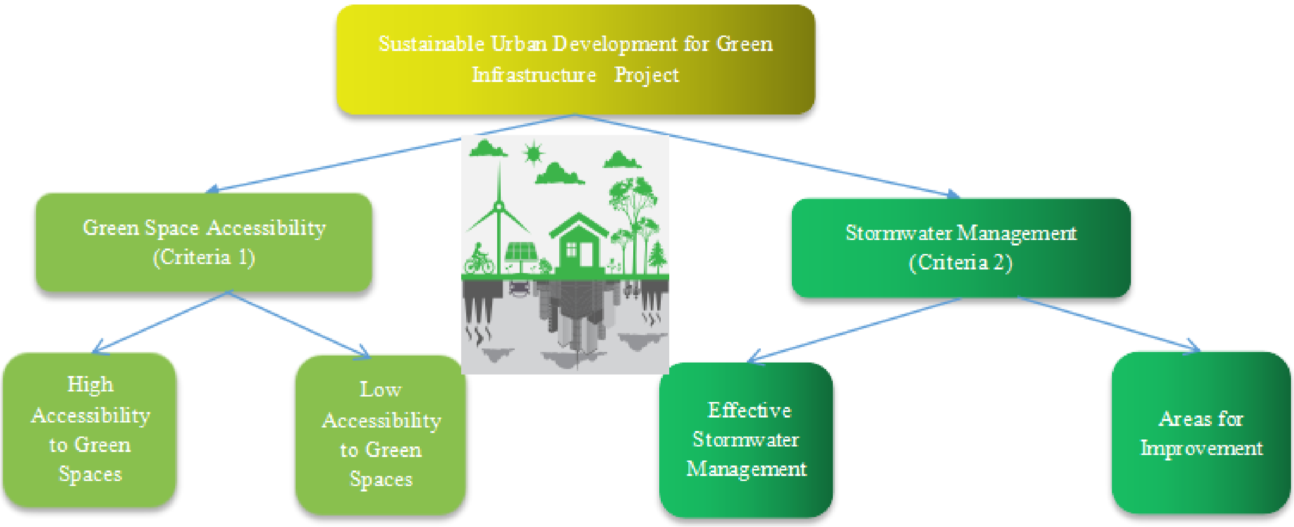

Fig. 1. Visual representation of the sustainable urban development for the Green Infrastructure Project, as illustrated in Example 3.2.

Taking into account the fundamental characteristics of Yager's operations $\oplus$ and $\otimes$, along with scalar multiplication and scalar power, as delineated in Theorem 3.1, we can observe their essential properties.

Theorem 3.1. *[16] Let $\mathcal{F}_1$ and $\mathcal{F}_2$ represent two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$. For any $\delta_1, \delta_2 > 0$, the following holds.*

(i). $\mathcal{F}_1 \oplus \mathcal{F}_2 = \mathcal{F}_2 \oplus \mathcal{F}_1$.

(ii). $\mathcal{F}_1 \otimes \mathcal{F}_2 = \mathcal{F}_2 \otimes \mathcal{F}_1$.

(iii). $\delta_1(\mathcal{F}_1 \oplus \mathcal{F}_2) = \delta_1\mathcal{F}_1 \oplus \delta_1\mathcal{F}_2$.

(iv). $(\delta_1 + \delta_2)\mathcal{F}_1 = \delta_1\mathcal{F}_1 \oplus \delta_2\mathcal{F}_1$.

(v). $(\mathcal{F}_1 \otimes \mathcal{F}_2)^{\delta_1} = \mathcal{F}_1^{\delta_1} \otimes \mathcal{F}_2^{\delta_1}$.

(vi). $\mathcal{F}_1^{\delta_1} \otimes \mathcal{F}_1^{\delta_2} = \mathcal{F}_1{}^{\delta_1+\delta_2}$.

Proof. The proof is easily shown by applying Definition 3.1. □

4. Innovative q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Aggregation Operator and Its Distinctive Features

This section presents two novel approaches, q-ROFGPOPRYWA and q-ROFGPOPRYWG, specifically designed for integrating q-Rung Orthopair Fuzzy Numbers (q-RLDFNs). To illustrate their practical utility, we showcase real-world instances related to the Sustainable Transportation Plan within urban development. We delve into fundamental characteristics such as idempotency, boundedness, and monotonicity.

4.1 Exploring the Aggregation Operator: q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average

In this section, we introduce the concept of AO, which we denote as q-ROFGPOPRYWA. We demonstrate that when aggregating m q-ROFNs, they retain their q-ROFN nature under the influence of AO, specifically q-ROFGPOPRYWA. We illustrate the practical application of AO, q-ROFGPOPRYWA, in urban development, particularly within the context of the Sustainable Transportation Plan (see Example 4.4). A visual representation of this application is provided. Furthermore, we thoroughly examine and provide simplified proofs for key properties of AO, q-ROFGPOPRYWA, such as Idempotency, Boundedness, and Monotonicity.

Definition 4.1 outlines AO, known as q-ROFGPOPRYWA, which involves integrating q-ROFNs, denoted as $\mathcal{F}_i$, using generalized power prioritization weighting coefficients η_i. These coefficients are determined by calculating power weighting coefficient factors, denoted as $\mathcal{U}_i$, and prioritized weighting coefficient factors, represented as $\mathcal{V}_i$, using specified equations. These equations incorporate parameters a and b within the range of $[0, 1]$.

Definition 4.1. *Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ represent q-Rung Orthopair Fuzzy numbers (q-ROFNs). Consider a weight vector $\mathfrak{W} = [\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_m]$ associated with the q-ROFNs $\mathcal{F}_i$, where $\mathfrak{w}_i > 0$ and $\sum_{i=1}^{m} \mathfrak{w}_i = 1$. The q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average (q-ROFGPOPRYWA) aggregation operator is then defined as:*

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i, \tag{5}$$

The Generalized Power Prioritization weighting coefficients η_i are computed as:

$$\eta_i = \frac{\mathfrak{w}_i \left[a(1 + \mathcal{U}_i) + b\mathcal{V}_i\right]}{\sum_{k=1}^{m} \mathfrak{w}_k \left[a(1 + \mathcal{U}_k) + b\mathcal{V}_k\right]}, \tag{6}$$

The calculation of the power weighting coefficient factors, denoted as $\mathcal{U}_i$, involves the following process:

$$\mathcal{U}_i = \sum_{\substack{k=1 \\ k \neq i}}^{m} \left[1 - \mathcal{D}(\mathcal{F}_i, \mathcal{F}_k)\right].$$

The determination of the prioritized weighting coefficient factors, represented as $\mathcal{U}_i$, entails the following procedure:

$$\mathcal{V}_1 = 1, \tag{7}$$

$$\mathcal{V}_i = \prod_{k=1}^{i-1} \mathcal{S}(\mathcal{F}_k), \quad i = 2, \ldots, m. \tag{8}$$

In this context, a and b belong to the interval $[0, 1]$ and serve as adjustable parameters according to specific needs. The power weighting coefficients, represented as $\mathcal{U}_i$, the prioritized weighting coefficients, denoted as $\mathcal{V}_i$, and the Generalized Power Prioritization weighting coefficients, referred to as η_i, remain constant throughout the entire article unless explicitly stated otherwise.

Substituting $a = 1$ and $b = 0$ into Equation (6)results in the transformed equation, referred to as Equation (5). This transformed equation adopts the structure of the q-Rung Orthopair Fuzzy Power Yager Weighted Average (q-ROFPOYWA) Aggregation Operator, as specified in Definition 4.2.

Definition 4.2. *Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ represent q-Rung Orthopair Fuzzy numbers (q-ROFNs). Consider a weight vector $\mathfrak{W} = [\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_m]$ associated with the q-ROFNs $\mathcal{F}_i$, where $\mathfrak{w}_i > 0$ and $\sum_{i=1}^{m} \mathfrak{w}_i = 1$. The q-Rung Orthopair Fuzzy Power Yager Weighted Average (q-ROFPOYWA) Aggregation Operator is then defined as:*

$$\text{q-ROFPOYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \gamma_i \mathcal{F}_i,$$

The Power weighting coefficients γ_i are computed as:

$$\gamma_i = \frac{\mathfrak{w}_i(1 + \mathcal{U}_i)}{\sum_{k=1}^{m} \mathfrak{w}_k(1 + \mathcal{U}_k)},$$

Substituting $a = 0$ and $b = 1$ into Equation (6) results in the transformed equation, referred to as Equation (5). This transformed equation adopts the structure of the q-Rung Orthopair Fuzzy prioritized Yager Weighted Average (q-ROFPOYWA) AO, as specified in Definition 4.3.

Definition 4.3. *Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ represent q-Rung Orthopair Fuzzy Numbers (q-ROFNs). Consider a weight vector $\mathfrak{W} = [\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_m]$ associated with the q-ROFNs $\mathcal{F}_i$, where $\mathfrak{w}_i > 0$ and $\sum_{i=1}^{m} \mathfrak{w}_i = 1$. The q-Rung Orthopair Fuzzy Prioritized Yager Weighted Average (q-ROFPRYWA) AO is then defined as:*

$$\text{q-ROFPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \psi_i \mathcal{F}_i,$$

The Prioritized weighting coefficients ψ_i are computed as:

$$\psi_i = \frac{\mathfrak{w}_i \mathcal{V}_i}{\sum_{k=1}^{m} \mathfrak{w}_k \mathcal{V}_k},$$

The following Theorem 4.1 establishes the aggregated value of q-ROFNs as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, through the utilization of the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average (q-ROFGPOPRYWA) operator (Equation (5)).

Theorem 4.1. *The combined value of q-Rung Orthopair Fuzzy Numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, using the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Average (q-ROFGPOPRYWA) operator, also results in a q-ROFN. The aggregation process is outlined as follows:*

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^{m} \eta_i (1 - \alpha_i^q)^\mu)^{1/\mu}\} \right]^{1/q}, \left[\min\{1, (\sum_{i=1}^{m} \eta_i (\beta_i^q)^\mu)^{1/\mu}\} \right]^{1/q} \right) \tag{9}$$

Proof. Mathematical induction offers a dependable approach to confirming the validity of a proof.

Case 1. Let us suppose m equals 2, and denote $\eta_1 \mathcal{F}_1$ as $(\bar{\alpha_1}, \bar{\beta_1})$, and $\eta_2 \mathcal{F}_2$ as $(\bar{\alpha_2}, \bar{\beta_2})$. In accordance with Equation (3.1), we can then express the following relationship:

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2) = \oplus_{i=1}^{2} \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, ((1 - \bar{\alpha_1}^q)^\mu + (1 - \bar{\alpha_2}^q)^\mu)^{1/\mu}\} \right]^{1/q}, \left[\min\{(1, ((\bar{\beta_1}^q)^\mu + (\bar{\beta_2}^q)^\mu)^{1/\mu}\} \right]^{1/q} \right) \tag{10}$$

It is evident that for each $i = 1, 2$, based on Equation (3), the following relations hold.

$$\bar{\mathfrak{T}}_i = \left[1 - \min\{1, (\eta_i (1 - \mathfrak{T}_i^q)^\mu)^{1/\mu}\} \right]^{1/q} \quad \text{for all} \quad \mathfrak{T} = \alpha$$

$$\bar{\mathfrak{T}}_i = \left[\min\{1, (\eta_i(\mathfrak{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for all} \quad \mathfrak{T} = \beta$$

Substituting these expressions into Equation 10, we obtain the modified form of Equation 10.

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2) = \oplus_{i=1}^2 \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^{2} \eta_i (1-\alpha_i^q)^\mu)^{1/\mu}\}\right]^{1/q}, \left[\min\{1, (\sum_{i=1}^{2} \eta_i (\beta_i^q)^\mu)^{1/\mu}\}\right]^{1/q}\right)$$

Case 2. If the result holds for $m = p$, we can establish its validity for $m = p+1$. Let us assume that $\oplus_{i=1}^p \eta_i \mathcal{F}_i = (\bar{\alpha}_\Delta, \bar{\beta}_\Delta)$ and $\eta_{p+1}\mathcal{F}_{p+1} = (\bar{\alpha}_{p+1}, \bar{\beta}_{p+1})$. Then, in accordance with Equation (3.1), we obtain:

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_p, \mathcal{F}_{p+1}) = \oplus_{i=1}^p \eta_i \mathcal{F}_i \oplus \eta_{p+1}\mathcal{F}_{p+1}$$

$$\left(\left[1 - \min\{1, ((1-\bar{\alpha}_\Delta^q)^\mu + (1-\bar{\alpha}_{p+1}^q)^\mu)^{1/\mu}\}\right]^{1/q}, \left[\min\{(1, ((\bar{\beta}_\Delta^q)^\mu + (\bar{\beta}_{p+1}^q)^\mu)^{1/\mu}\}\right]^{1/q}\right)$$

It is evident that from the assumption, we have the following.

$$\bar{\mathcal{T}}_\Delta = \left[1 - \min\{1, (\sum_{i=1}^{p} \eta_i (1-\mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for all} \quad \mathcal{T} = \alpha$$

$$\bar{\mathcal{T}}_\Delta = \left[\min\{1, (\sum_{i=1}^{p} \eta_i (\mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for all} \quad \mathcal{T} = \beta$$

Again, from Equation (3), we have the following.

$$\bar{\mathcal{T}}_{p+1} = \left[1 - \min\{1, (\eta_{p+1}(1-\mathcal{T}_{p+1}^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for all} \quad \mathcal{T} = \alpha$$

$$\bar{\mathcal{T}}_{p+1} = \left[\min\{1, (\eta_{p+1}(\mathcal{T}_{p+1}^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for all} \quad \mathcal{T} = \beta$$

If we substitute these values into Equation 2, then Equation 2 becomes as follows.

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_{p+1}) = \oplus_{i=1}^{p+1} \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^{p+1} \eta_i (1-\alpha_i^q)^\mu)^{1/\mu}\}\right]^{1/q}, \left[\min\{1, (\sum_{i=1}^{p+1} \eta_i (\beta_i^q)^\mu)^{1/\mu}\}\right]^{1/q}\right)$$

Therefore, the result remains valid for all values of m.

□

Example 4.4. *The* ***Sustainable Transportation Plan(STP)*** *[31] plays a crucial role in urban development, exerting significant influence on environmental sustainability and public welfare. Serving as a comprehensive guide, it ensures that transportation initiatives are executed effectively, aiming to mitigate environmental impact while improving overall urban livability. As urban planners evaluate this plan, their focus centers on specific criteria pertaining to sustainable transportation and its impact on the community.*

(i) ***Emission Reduction (Criteria 1)[32]:*** *A key objective of the STP is to minimize emissions from transportation activities, thereby improving air quality and mitigating climate change effects. By optimizing transportation protocols, urban planners aim to reduce vehicle emissions, positively impacting environmental and public health outcomes. Furthermore, a significant reduction in emissions not only enhances local air quality but also contributes to global efforts to combat climate change, reinforcing the overall sustainability of urban transportation systems.*

(ii) ***Mode Share Shift (Criteria 2)[33]:****Promoting a shift towards sustainable modes of transportation, such as walking, cycling, and public transit, is essential for reducing traffic congestion and dependence on fossil fuels. Evaluating the effectiveness of the STP in encouraging mode share shifts is crucial for optimizing urban mobility and reducing greenhouse gas emissions. Furthermore, a higher mode share for sustainable transportation options enhances community well-being by providing equitable access to affordable and environmentally friendly transportation alternatives.*

(iii) ***Accessibility and Equity (Criteria 3)[34]:*** *Assessing the accessibility of transportation services and ensuring equity in transportation planning is fundamental for promoting social inclusion and enhancing urban mobility for all residents. This criterion evaluates the STP's effectiveness in providing equitable access to transportation options, particularly for marginalized communities and underserved areas. Improving accessibility and equity in transportation not only enhances quality of life but also fosters economic development and social cohesion within urban areas.*

To demonstrate the application of an assessment framework, attention is focused on three critical criteria: Emission Reduction, Mode Share Shift, and Accessibility and Equity. However, it's important to note that evaluating the sustainable transportation plan should extend beyond these criteria. Utilizing advanced methodologies and leveraging data analytics specifically tailored for urban planning, these criteria can be quantified to ensure a comprehensive assessment of the plan's effectiveness in promoting sustainable urban transportation.

For Criteria 1, Emission Reduction, represented by $\mathcal{F}_1$ = (0.7, 0.3), the membership grade 0.7 signifies a high likelihood of achieving significant emission reductions, while the non-membership grade 0.3 denotes a lower likelihood.

Similarly, for Criteria 2, Mode Share Shift, represented by $\mathcal{F}_2$ = (0.8, 0.2). A membership grade of 0.8 indicates a moderate-to-high likelihood of shifting mode shares towards sustainable options, while a non-membership grade of 0.2 suggests a moderate-to-low likelihood.

For Criteria 3, Accessibility and Equity, represented by $\mathcal{F}_3$ = (0.9, 0.8), the membership grade 0.9 signifies a high likelihood of improving accessibility and equity in transportation, while the non-membership grade 0.8 denotes a moderate likelihood.

Assuming weights $\mathfrak{w}_1 = 0.2$, $\mathfrak{w}_2 = 0.3$, and $\mathfrak{w}_3 = 0.5$ for $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$ respectively, these weights and fuzzy numbers are applied solely for illustrating the application of AO, q-ROFGPOPRYWA (Equation (9)) in the environment of q-ROFNs. They do not represent real evaluation. To assess the efficacy of the sustainable transportation plan, the AO, q-ROFGPOPRYWA as defined in Equation (9) can be employed with the parameters $\mu = 3$, $a = 0.2$, and $b = 0.8$. Initially, calculate the power weighting coefficient factor denoted as $\mathcal{U}_1$, defined in (2.8).

$$\mathcal{U}_1 = \sum_{\substack{k=1 \\ k\neq 1}}^{3} [1 - \mathcal{D}(\mathcal{F}_1, \mathcal{F}_k)] = [1 - \mathcal{D}(\mathcal{F}_1, \mathcal{F}_2)] + [1 - \mathcal{D}(\mathcal{F}_1, \mathcal{F}_3)] \tag{11}$$

Here, the computation of $\mathcal{D}(\mathcal{F}_1, \mathcal{F}_2)$ involves utilizing the provided data and applying Equation

$$\begin{aligned}\mathcal{D}(\mathcal{F}_1, \mathcal{F}_2) &= \frac{1}{2}(|\alpha_1^q - \alpha_2^q| + |\beta_1^q - \beta_2^q|) \\ &= \frac{1}{2}(|0.7^4 - 0.8^4| + |0.3^4 - 0.2^4|) \\ &= 0.088000\end{aligned}$$

Similarly, the distance $\mathcal{D}(\mathcal{F}_1, \mathcal{F}_3)$ is calculated to be 0.4087500. Substituting these values into Equation (11), we find $\mathcal{U}_1 = 1.50325$. Likewise, we can determine $\mathcal{U}_2 = 1.58475$ and $\mathcal{U}_3 = 1.264$ using the same approach. From Equation (7), the prioritized weighting coefficient factor $\mathcal{V}_1 = 1$. Furthermore, the prioritized weighting coefficient factors $\mathcal{V}_i$ for $i = 2, 3$ can be calculated as defined in Equation (8).

$$\mathcal{V}_2 = \prod_{k=1}^{1} \mathcal{S}(\mathcal{F}_k) = \mathcal{S}(\mathcal{F}_1) \tag{12}$$

$$\mathcal{V}_3 = \prod_{k=1}^{2} \mathcal{S}(\mathcal{F}_k) = \mathcal{S}(\mathcal{F}_1)\mathcal{S}(\mathcal{F}_2) \tag{13}$$

In this context, the calculation of $\mathcal{S}(\mathcal{F}_1)$ involves utilizing the provided data and applying Equation (1):

$$\begin{aligned}\mathcal{S}(\mathcal{F}_1) &= \frac{1}{2} + \frac{1}{2}(\alpha_1^q - \beta_1^q) \\ &= \frac{1}{2} + \frac{1}{2}(0.7^4 - 0.3^4) \\ &= 0.616\end{aligned}$$

In a similar manner, we can compute $\mathcal{S}(\mathcal{F}_2) = 0.7040000$. Substituting these values into Equation (12) and Equation (13), we obtain $\mathcal{V}_2 = 0.616$ and $\mathcal{V}_3 = (0.616)(0.7040000) = 0.433664$. Now, let us calculate the Generalized Power Prioritization weighting coefficient η_1 using Equation (14) as follows.

$$\eta_1 = \frac{\mathfrak{w}_1\left[a(1 + \mathcal{U}_1) + b\mathcal{V}_1\right]}{\sum_{k=1}^{3} \mathfrak{w}_k\left[a(1 + \mathcal{U}_k) + b\mathcal{V}_k\right]}, \tag{14}$$

Here, $\mathfrak{w}_1\left[a(1 + \mathcal{U}_1) + b\mathcal{V}_1\right] = (0.2)\left[(0.2)(1 + 1.50325) + (0.8)(1)\right] = 0.2601300$. Similarly, we can compute $\mathfrak{w}_2\left[a(1 + \mathcal{U}_2) + b\mathcal{V}_2\right] = 0.3029250$ and $\mathfrak{w}_3\left[a(1 + \mathcal{U}_3) + b\mathcal{V}_3\right] = 0.3998656$. Substitute these values into Equation (14), we get $\eta_1 = 0.2701469$. Similarly, we can calculate $\eta_2 = 0.3145898$ and $\eta_3 = 0.4152633$. Now, we can aggregate $\mathcal{F}_1, \mathcal{F}_2$, and $\mathcal{F}_3$ by utilizing AO, q-ROFGPOPRYWA (Equation (9)) as follows.

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3) = \oplus_{i=1}^{3} \eta_i \mathcal{F}_i = (\alpha, \beta) \tag{15}$$

Here, the value of α is determined using the provided data and Equation (9):

$$\begin{aligned}\alpha &= \left[1 - \min\left\{1, (\eta_1(1 - \alpha_1^q)^\mu + \eta_2(1 - \alpha_2^q)^\mu + \eta_3(1 - \alpha_3^q)^\mu)^{\frac{1}{\mu}}\right\}\right]^{\frac{1}{q}} \\ &= \left[1 - \min\left(1, ((0.2701469)(1 - 0.7^4)^3 + (0.3145898)(1 - 0.8^4)^3 + (0.4152633)(1 - 0.9^4)^3)^{\frac{1}{3}}\right)\right]^{\frac{1}{4}} \\ &= 0.8026375\end{aligned}$$

Similarly, the value for β in Equation (15) is determined through the utilization of Equation (9) along with the given data. The resulting AO, q-ROFGPOPRYWA (Equation (15)), is formulated as:

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3) = (0.8026375, 0.7435048)$$

Applying the score function expressed in Equation (1) yields the following effectiveness score.

$$\mathcal{S}(\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3)) = 0.5547206$$

Therefore, the assessment of the sustainable transportation plan's effectiveness, employing AO, q-ROFGPOPRYWA on q-ROFNs with $q = 4$, yields a value of 0.5547206. Enhanced visualization for the evaluation of the Sustainable Transportation Plan is illustrated in Figure 2, as elaborated in Example 4.4.

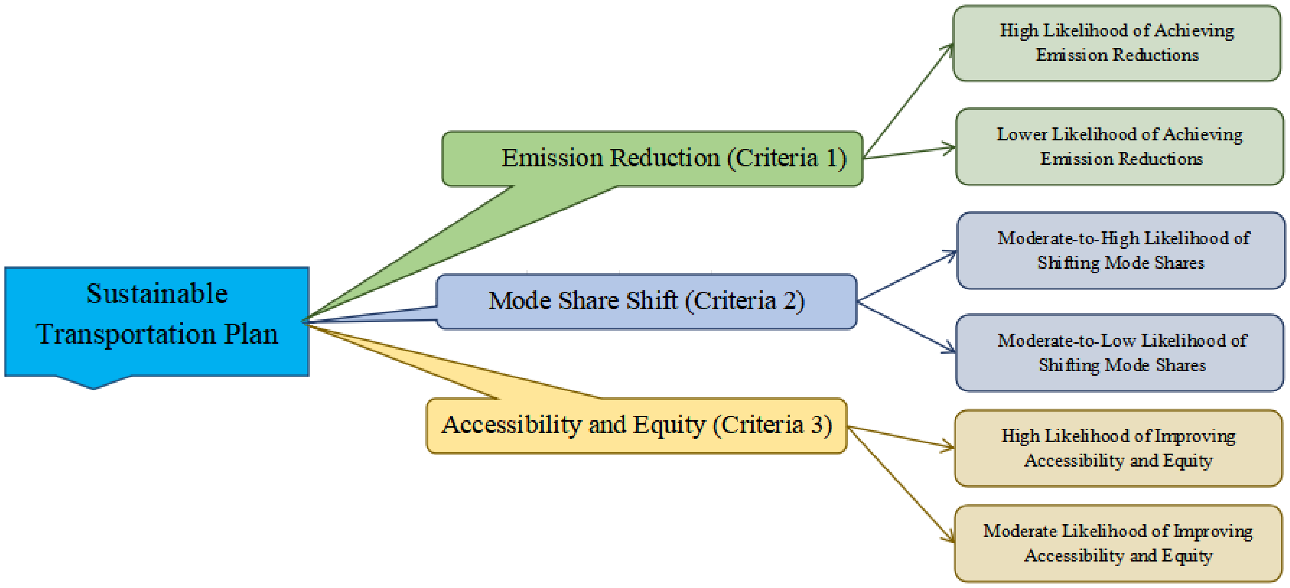

Fig. 2. Visualization depicting the evaluation of the Sustainable Transportation Plan, as discussed in Example 4.4

The Idempotency Property, as stated in Theorem 4.2, guarantees that employing the AO, $q\text{-}ROFGPOPRYWA$, on tuples $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$ to produce $\mathcal{F}$ consistently delivers identical outcomes. This underscores its reliability in maintaining fundamental structures intact.

Theorem 4.2. *(Idempotency Property) For any $1 \leq i \leq m$, let $\mathcal{F}_i = (\alpha_i, \beta_i)$, and suppose $\mathcal{F}_i = \mathcal{F}$, where $\mathcal{F} = (\alpha, \beta)$. If the aggregation operator $q\text{-}ROFGPOPRYWA$ is applied to $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$, the result is $\mathcal{F}$.*

Proof. If $\mathcal{F}_i$ is equal to $\mathcal{F}$, then Equation (9) can be expressed in a simplified form as:

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^{m} \eta_i (1-\alpha^q)^{\mu})^{1/\mu}\} \right]^{1/q}, \left[\min\{1, (\sum_{i=1}^{m} \eta_i (\beta^q)^{\mu})^{1/\mu}\} \right]^{1/q} \right), \tag{16}$$

As a result of the constraint $\sum_{i=1}^{m} \eta_i = 1$, Equation (16) undergoes a transformation to become (α, β). Thus,

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \mathcal{F}.$$

□

In Theorem 4.3, we establish bounds, denoted as $\mathcal{F}^-$ and $\mathcal{F}^+$, by considering the minimum and maximum values of parameters across a set of q-ROFNs. The theorem proves that when the q-ROFGPOPRYWA function is applied to these q-ROFNs, its output remains within these established bounds.

Theorem 4.3. *(Boundedness Property) Consider q-ROFNs represented by $\mathcal{F}_i = \langle(\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$. Assume that*

$$\mathcal{F}^- = \left(\min_i\{\alpha_i\}, \max_i\{\beta_i\}\right),$$
$$\mathcal{F}^+ = (\max_i\{\alpha_i\}, \min_i\{\beta_i\})$$

We establish the inequality: $\mathcal{F}^- \leq$ q-ROFGPOPRYWA$(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leq \mathcal{F}^+$.

Proof. Based on our assumption, the following inequalities hold for each index $i = 1, 2, \ldots, m$.

$$\min_i \mathcal{T}_i \leq \mathcal{T}_i \leq \max_i \mathcal{T}_i \quad \text{for} \quad \mathcal{T} = \alpha$$
$$\max_i \mathcal{T}_i \geq \mathcal{T}_i \geq \min_i \mathcal{T}_i \quad \text{for} \quad \mathcal{T} = \beta.$$

Therefore, the ensuing results are as follows:

$$\min_i \mathcal{T}_i \leq \left[1 - \min\{1, (\textstyle\sum_{i=1}^m \eta_i(1 - \mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \leq \max_i \mathcal{T}_i \quad \text{for} \quad \mathcal{T} = \alpha$$

$$\max_i \mathcal{T}_i \geq \left[\min\{1, (\textstyle\sum_{i=1}^m \eta_i(\mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \geq \min_i \mathcal{T}_i \quad \text{for} \quad \mathcal{T} = \beta.$$

Referring to Equation (9) and the score function outlined in Equation (1), we derive the subsequent result.

$$\mathcal{S}(\mathcal{F}^-) \leq \mathcal{S}(\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m)) \leq \mathcal{S}(\mathcal{F}^+).$$

Hence, the proof is finalized in accordance with Definition 2.6. □

Theorem 4.4 establishes that if one set of q-ROFNs is element-wise less than or equal to another set, the corresponding order of AO, q-ROFGPOPRYWA values remains unchanged.

Theorem 4.4. *(Monotonicity Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, and $\mathcal{F}_i' = (\alpha_i', \beta_i')$ for $i = 1, 2, \ldots, m$ represent the q-ROFNs. If $\mathcal{F}_i \leq \mathcal{F}_i'$ for $i = 1, 2, \ldots, m$, then*

$$\textit{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leq \textit{q-ROFGPOPRYWA}(\mathcal{F}_1', \mathcal{F}_2', \ldots, \mathcal{F}_m').$$

Proof. If we consider $\mathcal{F}_i \leq \mathcal{F}_i'$ for $i = 1, 2, \ldots, m$, as specified by Equation (2.5), then we derive:

$$\mathcal{S}(\mathcal{F}_i) \leq \mathcal{S}(\mathcal{F}_i') \quad \text{for} \quad i = 1, 2, \ldots, m.$$

As a result, we obtain the subsequent inequalities:

$$\mathcal{T}_i \leq \mathcal{T}_i' \quad \text{for} \quad \mathcal{T} = \alpha$$
$$\mathcal{T}_i \geq \mathcal{T}_i' \quad \text{for} \quad \mathcal{T} = \beta.$$

These inequalities can be converted into the following set of inequalities.

$$\left[1-\min\{1,(\textstyle\sum_{i=1}^{m}\eta_i(1-\mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \leq$$

$$\left[1-\min\{1,(\textstyle\sum_{i=1}^{m}\eta_i(1-(\mathcal{T}_i^{'})^q)^\mu)^{1/\mu}\}\right]^{1/q} \quad \text{for} \quad \mathcal{T}=\alpha$$

$$\left[\min\{1,(\textstyle\sum_{i=1}^{m}\eta_i(\mathcal{T}_i^q)^\mu)^{1/\mu}\}\right]^{1/q} \geq \left[\min\{1,(\textstyle\sum_{i=1}^{m}\eta_i(\mathcal{T}_i^{'})^q)^\mu)^{1/\mu}\right]^{1/q} \quad \text{for} \quad \mathcal{T}=\beta$$

Using Equation (9) and the score function defined in Equation (2.5), we obtain the following outcome.

$$\mathcal{S}(\text{q-ROFGPOPRYWA}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m)\leq\mathcal{S}(\text{q-ROFGPOPRYWA}(\mathcal{F}_1^{'},\mathcal{F}_2^{'},\ldots,\mathcal{F}_m^{'}).$$

Hence, the verification is concluded relying on Definition 2.6. □

4.2 q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Geometric Aggregation Operator

In this section, we introduce the concept of AO, represented as q-ROFGPOPRYWG. We establish that when m q-ROFNs are aggregated, the resulting entity remains a q-ROFN under the influence of AO, specifically q-ROFGPOPRYWG. The practical application of AO, denoted as q-ROFGPOPRYWG, is demonstrated in the realm of environmental studies, particularly within the framework of the Urban Agriculture and Food Security Plan (Example 4.8). A visual depiction of this application is presented. Furthermore, essential properties of AO, q-ROFGPOPRYWG, such as Idempotency, Boundedness, and Monotonicity, are thoroughly examined and supported by simplified proofs.

Definition 4.5 introduces AO, identified as q-ROFGPOPRYWG, which encompasses the amalgamation of q-ROFNs, labeled as $\mathcal{F}_i$, employing Generalized Power Prioritization weighting coefficients denoted as η_i. These coefficients are derived by computing power weighting coefficients factors, symbolized as $\mathcal{U}_i$, and prioritized weighting coefficients factors, represented as $\mathcal{V}_i$, through prescribed equations. These equations involve parameters a and b confined within the interval of $[0,1]$.

Definition 4.5. *Let $\mathcal{F}_i=(\alpha_i,\beta_i)$ for $i=1,2,\ldots,m$ represent q-Rung Orthopair Fuzzy Numbers (q-ROFNs). Consider a weight vector $\mathfrak{W}=[\mathfrak{w}_1,\mathfrak{w}_2,\ldots,\mathfrak{w}_m]$ associated with the q-ROFNs $\mathcal{F}_i$, where $\mathfrak{w}_i>0$ and $\sum_{i=1}^{m}\mathfrak{w}_i=1$. The q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Geometric (q-ROFGPOPRYWG) AO is then defined as:*

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m)=\otimes_{i=1}^{m}\eta_i\mathcal{F}_i, \tag{17}$$

When substituting $a=1$ and $b=0$ into Equation (14), we obtain the modified equation, denoted as Equation (17). This modified equation takes on the format of the q-Rung Orthopair Fuzzy Power Yager Weighted Geometric (q-ROFPOYWG) AO, outlined in Definition 4.6.

Definition 4.6. *Let $\mathcal{F}_i=(\alpha_i,\beta_i)$ for $i=1,2,\ldots,m$ represent q-Rung Orthopair Fuzzy Numbers (q-ROFNs). Consider a weight vector $\mathfrak{W}=[\mathfrak{w}_1,\mathfrak{w}_2,\ldots,\mathfrak{w}_m]$ associated with the q-ROFNs $\{_i$, where $\mathfrak{w}_i>0$ and $\sum_{i=1}^{m}\mathfrak{w}_i=1$. The q-Rung Orthopair Fuzzy Power Yager Weighted Geometric (q-ROFPOYWG) AO is then defined as:*

$$\text{q-ROFPOYWG}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m)=\otimes_{i=1}^{m}\gamma_i\mathcal{F}_i,$$

Substituting $a=0$ and $b=1$ into Equation (14) results in the transformed equation, referred to as Equation (17). This transformed equation adopts the structure of the q-Rung Orthopair Fuzzy prioritized Yager Weighted Geometric (q-ROFPRYWG) AO, as specified in Definition 4.7.

Definition 4.7. *Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ represent q-Rung Orthopair Fuzzy Numbers (q-ROFNs). Consider a weight vector $\mathfrak{w} = [\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_m]$ associated with the q-ROFNs $\mathcal{F}_i$, where $\mathfrak{w}_i > 0$ and $\sum_{i=1}^{m} \mathfrak{w}w_i = 1$. The q-Rung Orthopair Fuzzy Prioritized Yager Weighted Geometric (q-ROFPRYWG) AO is then defined as:*

$$\text{q-ROFPRYWG}(\mathcal{F}_1, \{_2, \ldots, \mathcal{F}_m) = \otimes_{i=1}^{m} \psi_i \mathcal{F}_i,$$

The subsequent Theorem 4.5 establishes the combined value of q-ROFNs as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, employing the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Geometric (q-ROFGPOPRYWG) operator (Equation (17)).

Theorem 4.5. *The combined value of q-Rung Orthopair Fuzzy Numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, using the q-Rung Orthopair Fuzzy Generalized Power Prioritized Yager Weighted Geometric (q-ROFGPOPRYWG) operator, also results in a q-ROFN. The aggregation process is outlined as follows:*

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i$$

$$\left(\left[\min\{1, (\sum_{i=1}^{m} \eta_i (\alpha_i^q)^{\mu})^{1/\mu}\} \right]^{1/q}, \left[1 - \min\{1, (\sum_{i=1}^{m} \eta_i (1 - \beta_i^q)^{\mu})^{1/\mu}\} \right]^{1/q} \right), \tag{18}$$

Proof. This can be demonstrated by employing the same methodologies as those utilized in Theorem 4.1. □

Example 4.8. (Urban Agriculture and Food Security Plan[35]) *The Urban Agriculture and Food Security Plan (UAFSP) is an essential framework for addressing food insecurity and promoting sustainable urban development. It aims to enhance food accessibility, improve nutritional outcomes, and foster community resilience through urban agriculture initiatives. Urban planners evaluate this plan based on specific criteria related to urban agriculture and its impact on food security.*

(i). ***Food Production Capacity (Criteria 1)[36]:*** *A key objective of the UAFSP is to increase local food production capacity, thereby reducing dependence on external food sources and enhancing food security within urban areas. By promoting urban agriculture practices such as rooftop gardening, community gardens, and vertical farming, urban planners aim to boost local food production. This criterion evaluates the effectiveness of the UAFSP in facilitating increased food production capacity, which is crucial for ensuring a reliable and sustainable food supply for urban residents.*

(ii). ***Access to Fresh and Nutritious Food (Criteria 2)[37]:*** *Promoting access to fresh, nutritious, and culturally appropriate food is essential for addressing food insecurity and improving public health outcomes. The UAFSP aims to enhance food access by supporting farmers' markets, community-supported agriculture (CSA) programs, and urban food distribution networks. Evaluating the effectiveness of the UAFSP in improving access to fresh and nutritious food options is crucial for enhancing food security and promoting healthy eating habits among urban residents.*

(iii). ***Community Engagement and Participation (Criteria 3)[38]:*** *Assessing community engagement and participation in urban agriculture initiatives is fundamental for promoting social inclusion and building community resilience. This criterion evaluates the UAFSP's effectiveness in fostering community involvement in food production, distribution, and decision-making processes. By engaging residents in urban agriculture activities such as community gardening and educational workshops, the UAFSP aims to strengthen social bonds and empower communities to address food security challenges collectively.*

(iv). ***Resource Efficiency and Sustainability (Criteria 4)[39]:*** *Assessing the resource efficiency and sustainability of urban agriculture practices is crucial for ensuring the long-term viability of food production systems. The UAFSP aims to promote sustainable agricultural techniques such as rainwater harvesting, composting, and agroecological farming methods. Evaluating the effectiveness of the UAFSP in enhancing resource efficiency and sustainability measures is essential for minimizing environmental impact and optimizing resource utilization in urban agriculture. By implementing sustainable practices, such as reducing water and energy consumption, minimizing waste generation, and promoting biodiversity, the UAFSP contributes to the resilience of urban food systems and mitigates ecological footprints.*

For Criteria 1, Food Production Capacity, represented by $\mathcal{F}_1 = (0.7, 0.3)$, a membership grade of 0.7 signifies a moderate-to-high likelihood of increasing food production capacity through urban agriculture initiatives, while a non-membership grade of 0.3 suggests a moderate-to-low likelihood. Similarly, for Criteria 2, Access to Fresh and Nutritious Food, represented by $\mathcal{F}_2 = (0.9, 0.4)$, a membership grade of 0.9 indicates a high likelihood of improving access to fresh and nutritious food options, while a non-membership grade of 0.4 denotes a lower likelihood. For Criteria 3, Community Engagement and Participation, represented by$\mathcal{F}_3 = (0.8, 0.1)$, a membership grade of 0.8 signifies a moderate likelihood of fostering community engagement and participation in urban agriculture initiatives, while a non-membership grade of 0.1 denotes a moderate likelihood. For Criteria 4, Resource Efficiency and Sustainability, represented by $\mathcal{F}_4 = (0.9, 0.3)$, a membership grade of 0.9 signifies a high likelihood of promoting resource efficiency and sustainability in urban agriculture practices, while a non-membership grade of 0.3 suggests a moderate likelihood.

Given the weights $\mathfrak{w}_1 = 0.1$, $\mathfrak{w}_2 = 0.3$, $\mathfrak{w}_3 = 0.2$, and $\mathfrak{w}_4 = 0.4$ assigned to $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, and $\mathcal{F}_4$ respectively, along with the q-ROFNs, it is important to note that these values are used solely for illustrative purposes in the context of AO, q-ROFGPOPRYWG (Equation (5)) within the framework of q-ROFNs ($q = 3$), and do not reflect real evaluation. To evaluate the effectiveness of urban agriculture and food security plan, we apply AO, q-ROFGPOPRYWG using the parameters $\mu = 3$, $a = 0.7$, and $b = 0.3$.

Firstly, we compute the power weighting coefficient factors, $\mathcal{U}_i$ for $i = 1, 2, 3, 4$, as defined in (2.8), following a similar procedure as in Example 4.4. These values are: $\mathcal{U}_1 = 2.498, \mathcal{U}_2 = 2.63, \mathcal{U}_3 = 2.641$, and $\mathcal{U}_4 = 2.667$.

Next, we determine the prioritized weighting coefficient factor $\mathcal{V}_1 = 1$ using Equation (7). Then, we compute $\mathcal{V}_i$ for $i = 2, 3, 4$ using Equation (8) with the same procedure used in Example 4.4, resulting in $\mathcal{V}_2 = 0.658$, $\mathcal{V}_3 = 0.547785$, and $\mathcal{V}_4 = 0.4138516$.

With the prioritized weighting coefficients calculated, we proceed to compute the Generalized Power Prioritization weighting coefficient η_i for $i = 1, 2, 3, 4$, using Equation (14) and a similar procedure as in Example 4.4. The resulting values are $\eta_1 = 0.1012223$, $\eta_2 = 0.30254$, $\eta_3 = 0.1998252$, and $\eta_4 = 0.3964125$.

Finally, we aggregate $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, and $\mathcal{F}_4$ using AO, q-ROFGPOPRYWG (Equation (17)).

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) = \otimes_{i=1}^{4} \eta_i \mathcal{F}_i = (\alpha, \beta) \tag{19}$$

In this context, we ascertain the value of α by utilizing the given data and referencing Equation (5).

$$\begin{aligned}\alpha &= \left[\min\{1, (\eta_1(\alpha_1^q)^\mu + \eta_2(\alpha_2^q)^\mu + \eta_3(\alpha_3^q + \eta_4(\alpha_4^q)^\mu)^{1/\mu}\}\right]^{1/q} \\ &= \left[1 - \min\left(1, \left((0.101)(1-0.7^3)^3 + (0.302)(1-0.9^3)^3 + (0.199)(1-0.8^3)^3 + (0.396)(1-0.9^3)^3\right)^{1/3}\right)\right]^{1/3} \\ &= 0.8753343\end{aligned}$$

Similarly, the values of β in Equation (19) are determined using Equation (17) and the provided data. The resulting AO, denoted as q-ROFGPOPRYWG (Equation (19)), can be expressed as follows:

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) = (0.8753343, 0.3190233)$$

By applying the score function presented in Equation (1), the corresponding effectiveness scores are obtained:

$$\mathcal{S}(\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4)) = 0.8515795$$

The effectiveness of urban agriculture and food security plan, utilizing AO q-ROFGPOPRYWG on q-ROFNs with $q = 3$, results in a value of 0.8515795. The Assessment Framework for urban agriculture and food security plan, discussed in Example 4.8, is illustrated in Figure 3.

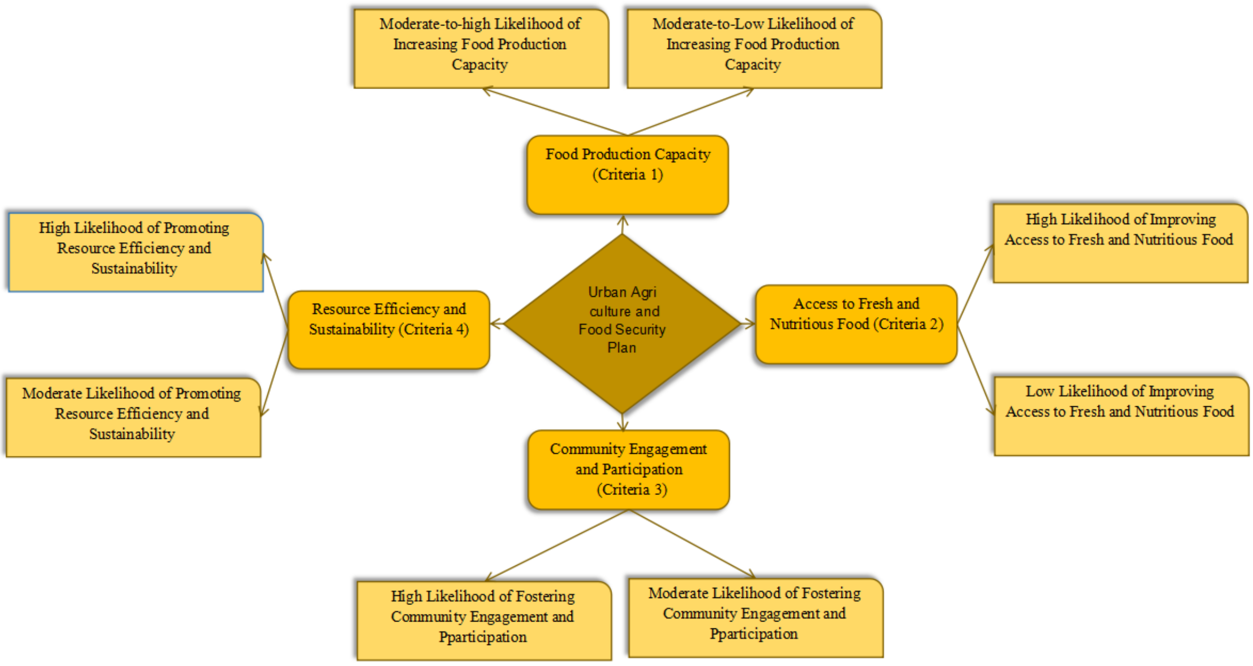

Fig. 3. Urban Agriculture and Food Security Plan discussed in Example 4.8

The Idempotency Property (Theorem 4.6) ensures that applying the AO, $q\text{-}ROFGPOPRYWG$ to tuples $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$ equivalent to $\mathcal{F}$ yields the same result, highlighting its stability in preserving underlying structures.

Theorem 4.6. *(Idempotency Property) For any $1 \leq i \leq m$, let $\mathcal{F}_i = (\alpha_i, \beta_i)$, and suppose $\mathcal{F}_i = \mathcal{F}$, where $\mathcal{F} = (\alpha, \beta)$. If the aggregation operator $q\text{-}ROFGPOPRYWG$ is applied to $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$, the result is $\mathcal{F}$.*

Proof. Demonstration of this can be established through employing the same methodologies as those employed in Theorem 4.2. □

Theorem 4.7. *(Boundedness Property) Consider q-ROFNs represented by $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$. Assume that*

$$\mathcal{F}^- = \left(\min_i\{\alpha_i\}, \max_i\{\beta_i\}\right)$$
$$\mathcal{F}^+ = \left(\max_i\{\alpha_i\}, \min_i\{\beta_i\}\right)$$

We establish the inequality: $\mathcal{F}^- \leq$ q-ROFGPOPRYWG$(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leq \mathcal{F}^+$.

Proof. Demonstration of this can be established through employing the same methodologies as those employed in Theorem 4.3. □

Theorem 4.8 establishes that if one set of q-ROFNs is element-wise less than or equal to another set, the corresponding AO, q-ROFGPOPRYWG values maintain the same order.

Theorem 4.8. *(Monotonicity Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, and $\mathcal{F}_i' = (\alpha_i', \beta_i')$ for $i = 1, 2, \ldots, m$ represent the q-ROFNs. If $\mathcal{F}_i \leq \mathcal{F}_i'$ for $i = 1, 2, \ldots, m$, then*

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leq \text{q-ROFGPOPRYWG}(\mathcal{F}_1', \mathcal{F}_2', \ldots, \mathcal{F}_m').$$

Proof. Demonstration of this can be established through employing the same methodologies as those employed in Theorem 4.4. □

5. Multi-Criteria Decision Making Methodology and Its Practical Application

The Multi-Criteria Decision Making (MCDM) Method comprises several key steps: identifying alternatives and criteria, assigning weights to each criterion, and employing q-Rung Orthopair Fuzzy Numbers to evaluate criteria against alternatives. The process then involves computing factor coefficients, such as power weighting and prioritized weighting coefficients. These coefficients are amalgamated into generalized power prioritized weighting coefficients, which are utilized for aggregating q-Rung Orthopair Fuzzy Numbers. Finally, a score value is computed for the aggregated information to determine the ranking of alternatives based on their performance against the specified criteria.

Step 1. Alternatives and Criteria Identification:

Consider a set of alternatives, $\mathscr{A}_1, \mathscr{A}_2, \ldots, \mathscr{A}_n$, and a set of criteria, $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$.

Step 2. Weight assigned to each criterion for identification:

Let $\mathfrak{w}_i$ represent the weight assigned to criteria $\mathcal{F}_i$, where $\mathfrak{w}_i > 0$ for $i = 1, 2, \ldots, m$, and $\sum_{i=1}^{m} \mathfrak{w}_i = 1$.

Step 3. Criterion-Alternative Evaluation using q-Rung Orthopair Fuzzy Numbers:

For each criterion $\mathcal{F}_i$ ($i = 1, 2, \ldots, m$), the evaluation against each alternative $\mathscr{A}_j$ ($j = 1, 2, \ldots, n$) is conducted utilizing q-Rung Orthopair Fuzzy Numbers. This is represented as $\mathcal{F}_{ij} = (\alpha_{ij}, \beta_{ij})$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Step 4. Compute the Factor of Power Weighting Coefficients:

The factor of power weighting coefficients $\mathcal{U}_{ij}$ (Equation (2.8)) are computed for every q-Rung Orthopair Fuzzy number $\mathcal{F}_{ij}$, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, in the following manner.

$$\mathcal{U}_{ij} = \sum_{\substack{k=1 \\ k \neq i}}^{m} [1 - \mathcal{D}(\mathcal{F}_{ij}, \mathcal{F}_{kj})] \tag{20}$$

In this context, the distance between $\mathcal{F}_{ij}$ and $\mathcal{F}_{kj}$ is computed using the following calculation.

$$\mathcal{D}(\mathcal{F}_{ij}, \mathcal{F}_{kj}) = \frac{1}{2} \left(|\alpha_{ij} - \alpha_{kj}| + |\beta_{ij} - \beta_{kj}| \right)$$

Step 5. Compute the Factor of Prioritized Weighting Coefficients:

The factor of prioritized weighting coefficients $\mathcal{V}_{1j}$ (Equation (7)) and $\mathcal{V}ij$ (Equation (8)) are computed for each q-Rung Orthopair Fuzzy number $\mathcal{F}_{ij}$, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, in the following manner.

$$\mathcal{V}_{1j} = 1, \tag{21}$$

$$\mathcal{V}_{ij} = \prod_{k=1}^{i-1} \mathcal{S}(\mathcal{F}_{kj}), \quad i = 2, \ldots, m. \tag{22}$$

In this context, the score of $\mathcal{F}_{kj}$ is computed using the following calculation.

$$\mathcal{S}(\mathcal{F}_{kj}) = \frac{1}{2} + \frac{1}{2}(\alpha_{kj}^q - \beta_{kj}^q)$$

Step 6. Compute the Factor of Generalized Power Prioritized Weighting Coefficients:

Formulate the factor of generalized power prioritized weighting coefficients η_{ij} by incorporating the criteria weights $\mathfrak{w}_i$ (where i ranges from 1 to m), along with parameters a and b constrained to the interval $[0, 1]$ for each q-Rung Orthopair Fuzzy number $\mathcal{F}_{ij}$, where $i = 1, 2, \ldots, m$ and $j = 2, \ldots, n$. This formulation integrates the factor of power weighting coefficients $\mathcal{U}_{ij}$ and the factor of prioritized weighting coefficients $\mathcal{V}_{ij}$.

$$\Gamma_{ij} = \mathfrak{w}_i \left[a(1 + \mathcal{U}_{ij}) + b\mathcal{V}_{ij}\right] \tag{23}$$

Step 7. Compute the Generalized Power Prioritized Weighting Coefficients:

The generalized power prioritized weighting coefficients η_{ij} (Equation (14)) are computed for each q-Rung Orthopair Fuzzy number $\{_{ij}$, with $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, utilizing Equation (23). The calculation is expressed by:

$$\eta_{ij} = \frac{\Gamma_{ij}}{\sum_{i=1}^m \Gamma_{ij}} \tag{24}$$

Step 8. Aggregate q-Rung Orthopair Fuzzy Numbers:

Aggregate each grouping of q-Rung Orthopair Fuzzy numbers $\mathcal{F}_{1j}, \mathcal{F}_{2j}, \ldots, \mathcal{F}_{mj}$ $(j = 1, 2, \ldots, n)$ using either q-ROFGPOPRYWA (Equation (9)) or q-ROFGPOPRYWG (Equation (18)).

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^m \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^m \eta_i (1 - \alpha_i^q)^\mu)^{1/\mu}\} \right]^{1/q}, \left[\min\{1, (\sum_{i=1}^m \eta_i (\beta_i^q)^\mu)^{1/\mu}\} \right]^{1/q} \right) \tag{25}$$

(or)

$$\text{q-ROFGPOPRYWG}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^m \eta_i \mathcal{F}_i$$

$$\left(\left[\min\{1, (\sum_{i=1}^m \eta_i (\alpha_i^q)^\mu)^{1/\mu}\} \right]^{1/q}, \left[1 - \min\{1, (\sum_{i=1}^m \eta_i (1 - \beta_i^q)^\mu)^{1/\mu}\} \right]^{1/q} \right), \tag{26}$$

Step 9. Compute the Score Value:

The score value for the aggregated information in **Step Step 8.** (Equation (25) or Equation (26)) is computed using Equation (1).

Step 10. Determine the ranking of alternatives:

The ranking of alternatives $\mathscr{A}_j$ $(j = 1, 2, \ldots, n)$ is determined based on the score value computed in **Step Step 9.**.

Example 5.1. (**Sustainable Urban Development)**

Sustainable urban development remains a cornerstone in addressing the multifaceted challenges that modern cities face, including climate change, inequality, and infrastructure stress. To advance this agenda, cities are increasingly adopting a mix of inclusive and tech-enabled solutions. This example highlights four promising alternatives: Inclusive Urban Governance, Net-Zero Urban Districts, Urban Agriculture and Vertical Farming, and Digital Twin Urban Planning. The importance of each is discussed below:

(i). ***Inclusive Urban Governance ($\mathscr{A}_1$):*** *Inclusive Urban Governance ensures that diverse voices are represented in planning and decision making, especially those of marginalized communities. It strengthens democracy, accountability, and social equity in urban development processes.*

Key elements of Inclusive Urban Governance include:

- (a) ***Participatory Budgeting:*** *Citizens co-decide how a portion of the municipal budget is spent, improving transparency and aligning priorities with local needs.*
- (b) ***Decentralized Governance Structures:*** *Local councils and neighborhood assemblies enable more responsive governance and community oversight.*
- (c) ***Digital Civic Platforms:*** *Apps and online platforms allow residents to give feedback, track city services, and engage in urban policy dialogues.*
- (d) ***Equity Audits:*** *Policy and project reviews assess and address potential disparities in urban service delivery and access.*

(ii). ***Net-Zero Urban Districts ($\mathscr{A}_2$):*** *Net-Zero Urban Districts aim to produce as much energy as they consume annually through renewable energy, efficiency, and sustainable design. These districts are models for carbon-neutral urban growth.*

Core strategies of Net-Zero Districts include:

- (a) ***District Heating and Cooling Systems:*** *Shared infrastructure reduces emissions and increases energy efficiency across buildings.*
- (b) ***High-Performance Building Envelopes:*** *Super-insulated structures minimize heating and cooling demands.*
- (c) ***On-site Renewable Energy:*** *Solar, wind, or geothermal systems meet energy needs at the district scale.*
- (d) ***Net Energy Monitoring Systems:*** *Real-time dashboards enable residents and managers to track energy use and production.*

(iii). ***Urban Agriculture and Vertical Farming ($\mathscr{A}_3$):*** *Urban Agriculture and Vertical Farming address food security and ecological sustainability by localizing food production within the city. They reduce food miles, enhance green cover, and create job opportunities.*

Significant features include:

(a) ***Rooftop and Indoor Farms:*** *Use hydroponics, aeroponics, and LED lighting to produce food in dense urban areas.*

(b) ***Community Gardens:*** *Shared plots managed by residents foster food sovereignty and social cohesion.*

(c) ***Edible Landscapes:*** *Public spaces planted with fruit trees and vegetables integrate food production into daily urban life.*

(d) ***Waste-to-Fertilizer Systems:*** *Composting urban organic waste closes the nutrient loop and enriches urban soils.*

(iv). ***Digital Twin Urban Planning ($\mathscr{A}_4$):*** *Digital Twin Urban Planning leverages virtual models of cities that are continuously updated with real-time data. These twins simulate scenarios, improve planning accuracy, and enhance city management.*

Key attributes include:

(a) ***Real-Time Monitoring:*** *Sensor networks feed data into models for air quality, traffic, and energy flows.*

(b) ***Predictive Simulation:*** *Helps anticipate outcomes of zoning changes, infrastructure upgrades, and climate impacts.*

(c) ***Stakeholder Visualization Tools:*** *3D interfaces allow planners and residents to interactively explore future urban scenarios.*

(d) ***Integration with IoT and AI:*** *Combines smart infrastructure with data analytics for optimized urban systems.*

To address the Multi-Criteria Decision-Making (MCDM) challenge of selecting the most appropriate sustainable urban development alternative among Inclusive Urban Governance, Net-Zero Urban Districts, Urban Agriculture and Vertical Farming, and Digital Twin Urban Planning, it is essential to prioritize these options based on key criteria such as Participatory Governance, Carbon and Resource Efficiency, Urban Resilience and Adaptability, and Systems Innovation and Integration.

Step 1. Alternatives and Criteria Identification:

The alternatives denoted as $\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3,$ and $\mathscr{A}_4$ represent Sustainable Urban Development, specifically Inclusive Urban Governance, Net-Zero Urban Districts, Urban Agriculture and Vertical Farming, and Digital Twin Urban Planning respectively. The criteria $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3,$ and $\mathcal{F}_4,$ are established as assessment parameters for Sustainable Urban Development, namely, Participatory Governance, Carbon and Resource Efficiency, Urban Resilience and Adaptability, and Systems Innovation and Integration respectively.

Step 2. Weight assigned to each criterion for identification:

Let us assume that $\mathfrak{w}_1 = 0.2, \mathfrak{w}_2 = 0.3, \mathfrak{w}_3 = 0.1,$ and $\mathfrak{w}_4 = 0.4,$ represent the weights assigned to criteria $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4,$ respectively. It is important to note that these weights are not based on real values but are used for illustrative purposes in the context of the proposed Multi-Criteria Decision-Making (MCDM) method (refer to Section 5).

Step 3. Criterion-Alternative Evaluation using q-Rung Orthopair Fuzzy Numbers:

Each criterion $\mathcal{F}_i$ $(i = 1, 2, \ldots, 4)$ is assessed against every alternative $\mathscr{A}_j$ $(j = 1, 2, \ldots, 4)$ using the q-Rung Orthopair Fuzzy Number (q-ROFN) representation with $q = 3$, denoted as

$\mathcal{F}_{ij} = \langle(\alpha_{ij}, \beta_{ij})\rangle$ for $i = 1, 2, \ldots, 4$ and $j = 1, 2, \ldots, 4$. The tabular representation of q-ROFN is provided in Table 1. It is essential to emphasize that the values presented in Table 1 are hypothetical and are utilized for explanatory purposes within the framework of the proposed Multi-Criteria Decision-Making (MCDM) method (see Section 5).

Table 1
Tabular form of q-ROFNs (q=3)

$(\mathcal{F}/\mathscr{A})$	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
$\mathcal{F}_1$	(0.9,0.4)	(0.2,0.6)	(0.8,0.1)	(0.3,0.5)
$\mathcal{F}_2$	(0.9,0.3)	(0.4,0.8)	(0.4,0.2)	(0.2,0.2)
$\mathcal{F}_3$	(0.9,0.5)	(0.5,0.9)	(0.7,0.3)	(0.3,0.5)
$\mathcal{F}_4$	(0.8,0.2)	(0.3,0.6)	(0.7,0.4)	(0.3,0.4)

Step 4. Compute the Factor of Power Weighting Coefficients:

The factor of power weighting coefficients, denoted as $\mathcal{U}_{ij}$, are calculated for each q-ROFN $\mathcal{F}_{ij}$ (Table 1), where $i = 1, 2, \ldots, 4$ and $j = 1, 2, \ldots, 4$. Specifically, we focus on the calculation of $\mathcal{U}_{11}$, and the remaining coefficients can be determined in a similar fashion. Referring to Equation 20, we obtain:

$$\mathcal{U}_{11} = [1 - \mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{21})] + [1 - \mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{31})] + [1 - \mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{41})] \tag{27}$$

In this scenario, the computation of the distance between $\mathcal{F}_{11}$ and $\mathcal{F}_{21}$ is carried out as follows.

$$\mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{21}) = \frac{1}{2}\left(|\alpha_{11}^q - \alpha_{21}^q| + |\beta_{11}^q - \beta_{21}^q|\right) \tag{28}$$

Substituting the values of α_{i1}, β_{i1} (where $i = 1, 2$) from Table 1, along with $q = 3$, into Equation (28), yields $\mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{21}) = 0.0185$. Similarly, calculating the distances results in $\mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{31}) = 0.0305$, and $\mathcal{D}(\mathcal{F}_{11}, \mathcal{F}_{41}) = 0.1365$. By substituting these values into Equation (27), we find $\mathcal{U}_{11} = 2.8145$. Similarly, the values for $\mathcal{U}_{ij}$ can be computed for $i = 2, \ldots, 4$ and $j = 2, \ldots, 4$ using the same methodology. The resulting power weighting coefficients $\mathcal{U}_{ij}$ for each q-ROFN $\mathcal{F}_{ij}$ are summarized in Table 2.

Table 2
Factor of Power Weighting Coefficients $\mathcal{U}_{ij}$ for q-ROFN in Table 1

$(\mathcal{F}/\mathscr{A})$	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
$\mathcal{F}_1$	2.8145	2.4995	2.559	2.9015
$\mathcal{F}_2$	2.8145	2.5185	2.456	2.8265
$\mathcal{F}_3$	2.7535	2.2405	2.735	2.9015
$\mathcal{F}_4$	2.5785	2.5185	2.698	2.9015

Step 5. Compute the Factor of Prioritized Weighting Coefficients:

The factor of prioritized weighting coefficients, denoted as $\mathcal{V}_{ij}$, are computed for each q-ROFN $\mathcal{F}_{ij}$ (Table 1), where $i = 1, 2, \ldots, 4$ and $j = 1, 2, \ldots, 4$. Specifically, we focus on the computation of $\mathcal{V}_{11}$, $\mathcal{V}_{21}$, $\mathcal{V}_{31}$ and the remaining coefficients can be determined similarly. From the equation labeled as Equation 21, we obtain

$$\mathcal{V}_{11} = 1$$

By referring to Equation 22, it follows that

$$\mathcal{V}_{21} = \mathcal{S}(\mathcal{F}_{11}) = \frac{1}{2} + \frac{1}{2}(\alpha_{11}^q - \beta_{11}^q) \tag{29}$$

$$\mathcal{V}_{31} = \mathcal{S}(\mathcal{F}_{11})\mathcal{S}(\mathcal{F}_{21}) = (\frac{1}{2} + \frac{1}{2}(\alpha_{11}^q - \beta_{11}^q))(\frac{1}{2} + \frac{1}{2}(\alpha_{21}^q - \beta_{21}^q)) \tag{30}$$

Substituting the values of α_{i1}, β_{i1} (where $i = 1, 2$) from Table 1, alongside $q = 3$, into Equation (29) and Equation (30), we obtain $\mathcal{V}_{21} = 0.8325$ and $\mathcal{V}_{31} = 0.7084575$. Similarly, the values for $\mathcal{V}_{ij}$ can be computed for $i = 4$ and $j = 2, 3, 4$ using the same methodology. The resulting factor of prioritized weighting coefficients $\mathcal{V}_{ij}$ for each q-ROFN $\mathcal{F}_{ij}$ are detailed in Table 3.

Table 3
Factor of Prioritized Weighting Coefficients $\mathcal{V}_{ij}$ for q-ROFN in Table 1

$(\mathcal{F}/\mathscr{A})$	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
$\mathcal{F}_1$	1	1	1	1
$\mathcal{F}_2$	0.8325	0.396	0.7555	0.451
$\mathcal{F}_3$	0.7084575	0.109296	0.398904	0.2255
$\mathscr{F}_4$	0.5681829	0.0216406	0.2624788	0.1017005

Step 6. Compute the Factor of Generalized Power Prioritized Weighting Coefficients:

Assuming the parameters $a = 0.7$ and $b = 0.3$ in the interval $[0, 1]$, the factor of generalized power prioritized weighting coefficients, denoted as Γ_{ij}, are computed for each q-ROFN $\mathcal{F}_{ij}$ (Table 1), where $i = 1, 2, \ldots, 4$ and $j = 1, 2, \ldots, 4$. Specifically, we focus on the computation of Γ_{11}, and the remaining coefficients can be determined similarly. From referencing Equation 23, we derive:

$$\Gamma_{11} = \mathfrak{w}_1 \left[a(1 + \mathcal{U}_{11}) + b\mathcal{V}_{11}\right] \tag{31}$$

Replace $\mathfrak{w}_1$ with the weights of criteria obtained in **Step Step 2.**, $\mathcal{U}_{11}$ with the corresponding value from Table 2, and $\mathcal{V}_{11}$ with the respective value from Table 3 in Equation 31. This yields $\Gamma_{11} = 0.59403$. Similarly, for $i = 2, \ldots, 4$ and $j = 2, \ldots, 4$, Γ_{ij} values can be computed following the same procedure. The resulting factor of generalized power prioritized weighting coefficients Γ_{ij} for each q-ROFN $\mathcal{F}_{ij}$ are presented in detail in Table 4.

Table 4
Factor of Generalized Power Prioritized Weighting Coefficients Γ_{ij} for q-ROFN in Table 1

$(\mathcal{F}/\mathscr{A})$	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
$\mathcal{F}_1$	0.59403	0.54993	0.55826	0.60621
$\mathcal{F}_2$	0.87597	0.774525	0.793755	0.844155
$\mathcal{F}_3$	0.2839987	0.2301139	0.2734171	0.27987
$\mathcal{F}_4$	1.0701619	0.9877769	1.0669375	1.1046241

Step 7. Compute the Generalized Power Prioritized Weighting Coefficients:

The generalized power prioritized weighting coefficients η_{ij}, are calculated individually for each q-Rung Orthopair Fuzzy number $\mathcal{F}_{ij}$ (Table 1), where $i = 1, 2, \ldots, 4$ and $j = 1, 2, \ldots, 4$. Specifically, our focus is on determining the value of Γ_{11}, with the remaining coefficients following a similar process. Referring to Equation (24), we derive:

$$\eta_{11} = \frac{\Gamma_{11}}{\sum_{i=1}^{5} \Gamma_{i1}} \tag{32}$$

Insert the value of Γ_{i1} $(i = 1, 2, \ldots, 4)$ from Table 4 into Equation (32). Upon calculation, Equation (32) produces $\Gamma_{11} = 0.2103386$. Similarly, the values of Γ_{ij} can be computed for $i = 2, \ldots, 4$ and $j = 2, \ldots, 4$, following the same methodology. The resulting generalized power prioritized weighting coefficients η_{ij} for each q-Rung Orthopair Fuzzy number $\mathcal{F}_{ij}$ are detailed in Table 5.

Table 5
Generalized Power Prioritized Weighting Coefficients Γ_{ij} for q-ROFN in Table 1

$(\mathcal{F}/\mathscr{A})$	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
$\mathcal{F}_1$	0.2103386	0.2163081	0.2073489	0.2138413
$\mathcal{F}_2$	0.31017	0.3046498	0.2948165	0.2977767
$\mathcal{F}_3$	0.1005604	0.0905124	0.1015526	0.0987245
$\mathcal{F}_4$	0.378931	0.3885297	0.3962819	0.3896575

Step 8. Aggregate q-Rung Orthopair Fuzzy Numbers:

Aggregate the q-ROFNs, $\mathcal{F}_{1j}, \mathcal{F}_{2j}, \ldots, \mathcal{F}_{4j}$ $(j = 1, 2, \ldots, 4)$, provided in Table 1, utilizing either the AO, q-ROFGPOPRYWA (Equation (25)), or q-ROFGPOPRYWG (Equation (26)). Specifically, focus on aggregating $\mathcal{F}_{11}, \mathcal{F}_{21}, \ldots, \mathcal{F}_{41}$ using the AO, q-ROFGPOPRYWA (Equation (25)), with parameter $\mu = 3$, as follows.

$$\text{q-ROFGPOPRYWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i$$

$$\left(\left[1 - \min\{1, (\sum_{i=1}^{4} \eta_i (1 - \alpha_i^3)^3)^{1/3}\} \right]^{1/3}, \left[\min\{1, (\sum_{i=1}^{4} \eta_i (\beta_i^3)^3)^{1/3}\} \right]^{1/3} \right)$$

Substitute the values of η_{i1} $(i = 1, 2, \ldots, 4)$ from Table 5, and the values of α_{i1}, β_{i1} $(i = 1, 2, \ldots, 4)$ from Table 1 into Equation (25).
After computation, Equation (25) yields q-ROFGPOPRYWA$(\mathcal{F}_{11}, \mathcal{F}_{21}, \ldots, \mathcal{F}_{41})$ $= \langle 0.8510987, 0.3992658 \rangle$. Similarly, aggregation of q-ROFNs, $\mathcal{F}_{1j}, \mathcal{F}_{2j}, \ldots, \mathcal{F}_{4j}$ for $j = 2, \ldots, 4$ can be performed. Likewise, aggregation of q-ROFN, $\mathcal{F}_{1j}, \mathcal{F}_{2j}, \ldots, \mathcal{F}_{4j}$ $(j = 1, 2, \ldots, 4)$, as provided in Table 1, can be carried out using the AO, q-ROFGPOPRYWG (Equation (26)). The resulting aggregation values are displayed in Table 6.

Step 9. Compute the Score Value:

The scores for the aggregated information in Table 6 from **Step 8.** are computed using Equation (1). These calculated scores are displayed in Table 7.

Table 6
Aggregation values for q-ROFNs (Table 1) with proposed operators

Alternative	q-ROFGPOPRYWA Operator	q-ROFGPOPRYWG Operator
$\mathscr{A}_1$	$\langle 0.8510987, 0.3992658 \rangle$	$\langle 0.8719892, 0.3305903 \rangle$
$\mathscr{A}_2$	$\langle 0.3473038, 0.7574304 \rangle$	$\langle 0.4003814, 0.6767071 \rangle$
$\mathscr{A}_3$	$\langle 0.6373476, 0.3617282 \rangle$	$\langle 0.7136214, 0.310282 \rangle$
$\mathscr{A}_4$	$\langle 0.2770477, 0.4470214 \rangle$	$\langle 0.2887968, 0.4003286 \rangle$

Table 7
Score values for aggregated information given in Table 6

Proposed Operator	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
q-ROFGPOPRYWA	0.7259164	0.2949367	0.6378097	0.4150131
q-ROFGPOPRYWG	0.7706994	0.3618372	0.7016697	0.4442341

Step 10. Determine the ranking of alternatives:

The ranking of alternatives $\mathscr{A}_j$ $(j = 1, 2, \ldots, 4)$ is determined based on the score values computed in **Step 9.** Referring to Table 7, the ranking of alternatives $\mathscr{A}_j$ $(j = 1, 2, \ldots, 4)$ is $\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$ according to both the proposed aggregation operators, q-ROFGPOPRYWA and q-ROFGPOPRYWG.

5.1 *Analyzing the Sensitivity of Aggregation Operators under Fixed q and Varying μ Parameters*

The provided table (Table 8) offers a comprehensive sensitivity analysis of two proposed aggregation operators, "q-ROFGPOPRYWA"and "q-ROFGPOPRYWG", across different parameter configurations (q, a, b, μ). Each row in the tables represents a specific parameter setting, showcasing how these operators perform across four criteria ($\mathscr{A}_1$ to $\mathscr{A}_4$). For instance, in the setting where $q = 3$, $a = 1$, $b = 0$, and $\mu = 3$, "q-ROFGPOPRYWA"achieves scores of 0.7256219 , 0.2952278 , 0.6376156 , 0.4157176 for $\mathscr{A}_1$ to $\mathscr{A}_4$, respectively, while "q-ROFGPOPRYWG"achieves scores of 0.7708255, 0.3621088, 0.6995772, 0.4448052 under the same configuration. These scores provide insights into each operator's performance across different criteria and parameter combinations, considering weights $\mathfrak{w}_i = [0.2, 0.3, 0.1, 0.4]$ (where i ranges from 1 to 4) for each criterion. The "Ranking"column in the tables offers a preference order based on these scores, indicating the effectiveness of each operator under varying conditions. A higher ranking suggests better performance, providing valuable insights into the suitability of "q-ROFGPOPRYWA"and "q-ROFGPOPRYWG"for decision-making tasks. This ranking helps identify the most effective operator for different parameter settings and criteria, aiding in making informed decisions regarding the selection of aggregation operators. The graphical view of the sensitivity analysis is depicted in Figure 4 .

Table 8
Sensitivity analysis explored proposed aggregation operators across μ, a, b, with fixed $q = 3$.

q	a	b	μ	Proposed Operator	Score Values $\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$	Ranking
3	1	0	3	q-ROFGPOPRYWA	0.7256219	0.2952278	0.6376156	0.4157176	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.7708255	0.3621088	0.6995772	0.4448052	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0.7	0.3	4	q-ROFGPOPRYWA	0.7135452	0.2848023	0.6261368	0.4099703	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.7731602	0.3753047	0.7081246	0.4466786	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0.2	0.8	100	q-RLDFGPOPRYWA	0.6532524	0.1854934	0.5109691	0.3789703	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.8204204	0.4460693	0.792438	0.5150883	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0	1	1000	q-ROFGPOPRYWA	0.9001512	0.1526421	0.7008973	0.6042772	$\mathscr{A}_1 \succ \mathscr{A}_3 1 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.8452495	0.1998247	0.8363701	0.3957228	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$

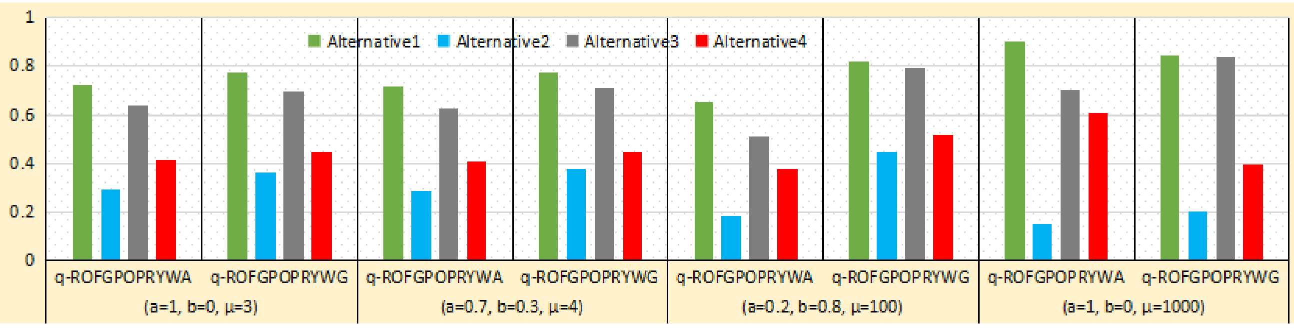

Fig. 4. Scores behavior of alternatives for different μ and fixed $q = 4$

5.2 *Analyzing the Sensitivity of Aggregation Operators under Fixed μ and Varying q Parameters*

The provided table (Table 9) and accompanying text present a sensitivity analysis of two proposed aggregation operators, “q-ROFGPOPRYWA”and “q-ROFGPOPRYWG”, across different parameter configurations (q, a, b) with a fixed value of $\mu = 4$. Each row in the table represents a specific parameter setting, and the “Score Values”columns depict the performance of the operators across four criteria ($\mathscr{A}_1$ to $\mathscr{A}_4$). For instance, under the setting where $q = 7$, $a = 1$, $b = 0$, and $\mu = 4$, “q-ROFGPOPRYWA”achieves scores of 0.6972418, 0.278535, .6518909, 0.4035534 for $\mathscr{A}_1$ to $\mathscr{A}_4$, respectively, while “q-ROFGPOPRYWG”achieves scores of 0.7512792, 0.3713738, 0.7014765 and 0.4301778 under the same configuration. These scores provide insights into each operator's performance across different criteria and parameter combinations, considering weights $\mathfrak{w}_i = [0.2, 0.1, 0.2, 0.3, 0.2]$ (where i ranges from 1 to 4) for each criterion. These scores reflect the relative performance of the operators under different parameter combinations and criteria with ranking . The “Ranking”column offers a preference order based on these scores, indicating the effectiveness of each operator. A higher ranking suggests better performance, with operators ranked from most to least effective based on their scores across the criteria, providing valuable insights into their suitability for decision-making tasks under varying conditions.The graphical view sensitivity analysis is given in Figure 5

Table 9
Sensitivity analysis explored proposed aggregation operators across q, a, b, with fixed $\mu = 4$.

μ	a	b	q	Proposed Operator	Score Values				Ranking
					$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$	
4	1	0	7	q-ROFGPOPRYWA	0.6972418	0.278535	0.6518909	0.4035534	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.7512792	0.3713738	0.7014765	0.4301778	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0.7	0.3	25	q-ROFGPOPRYWA	0.6969732	0.2881966	0.6789159	0.4007726	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ A_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.7198459	0.3332042	0.7012722	0.410858	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0.2	0.8	100	q-ROFGPOPRYWA	0.6998119	0.0527295	0.6955258	0.2506233	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
				q-ROFGPOPRYWG	0.9495733	0.3093027	0.8987316	0.6498578	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
	0	1	5000	q-ROFGPOPRYWA	0.5	0.5	0.5	0.5	$\mathscr{A}_1 = \mathscr{A}_3 = \mathscr{A}_4 = \mathscr{A}_2$
				q-ROFGPOPRYWG	0.5	0.5	0.5	0.5	$\mathscr{A}_1 = \mathscr{A}_3 = \mathscr{A}_4 = \mathscr{A}_2$

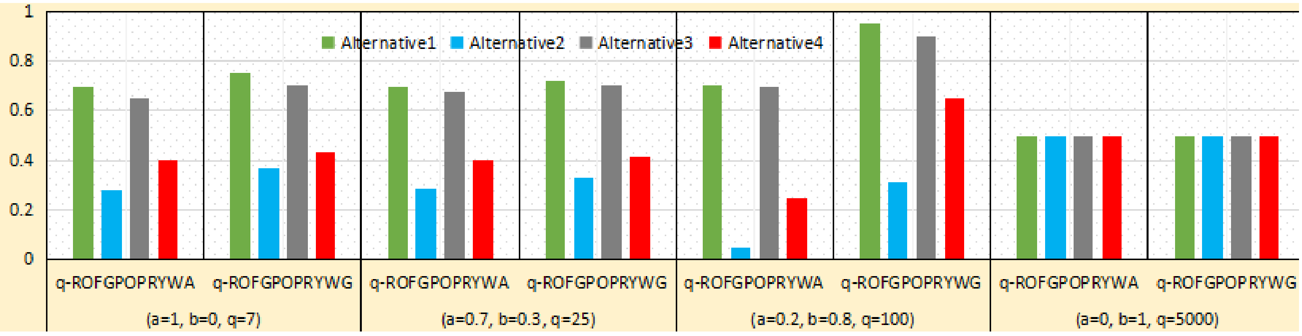

Fig. 5. Scores behavior of alternatives for different q and fixed $\mu = 4$

5.3 *Comparative Evaluation of the Proposed Aggregation Operators*

In this segment, we evaluate the presented model by comparing it with existing models to assess the effectiveness and superiority of the proposed aggregation technique. The results in Table 10 indicate that the most favorable alternative among all the discussed operators is denoted as $\mathscr{A}_1$ affirming the consistency and credibility of the proposed model. While both the presented and existing models yield identical optimal solutions for a Multiple Criteria Decision Making (MCDM) problem, the Generalized Power Prioritized Yager Weighted operators exhibit sensitivity to extreme values. This sensitivity enables them to capture noteworthy changes in input variables, proving beneficial in situations where specific variables exert a disproportionate influence on the overall aggregation. The integration of power and prioritized Operators establishes a versatile framework capable of adapting to diverse contexts and data distributions.

Table 10
Score and ranking of the aggregated q-ROFN for $\mathscr{A}_j$ $(j = 1, \ldots, 4)$ with $q = 3$ and $\mu = 5$

Operator	Score Value				Ranking
	$\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$	
Frank Weighted Averaging (q-ROFFWA) [7]	0.6050	0.4531	0.5482	0.4964	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Frank Weighted Geometric (q-ROFFWG) [7]	0.6000	0.4428	0.5342	0.4921	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Yager Weighted Averaging (Yq-ROFWA) [16]	0.1521	0.0003	0.0304	0.0000	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Yager Weighted Geometric (Yq-ROFWG) [16]	0.5184	0.4373	0.5055	0.4995	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Einstein Weighted Averaging (q-ROFEWA) [8]	0.8154	0.3600	0.6438	0.4894	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Einstein Weighted Geometric (q-ROFEWG) [8]	0.8003	0.3287	0.6013	0.4766	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Aczel-Alsina Weighted Averaging (q-ROFAAWA) [29]	0.8397	0.4160	0.7051	0.5021	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Aczel-Alsina Weighted Geometric (q-ROFAAWG) [29]	0.7444	0.2251	0.5304	0.4578	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Schweizer-Sklar Power Weighted Averaging (q-ROFSSWA) [40]	0.7811	0.2718	0.6199	0.4611	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Schweizer-Sklar Power Weighted Geometric (q-ROFSSWG) [40]	0.8180	0.3622	0.6812	0.4798	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Generalized Power Prioritized Yager Weighted Average (q-ROFGPOPRYWA)	0.7038	0.2770	0.6167	0.4069	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$
Generalized Power Prioritized Yager Weighted Geometric (q-ROFGPOPRYWG)	0.7756	0.3861	0.7109	0.4492	$\mathscr{A}_1 \succ \mathscr{A}_3 \succ \mathscr{A}_4 \succ \mathscr{A}_2$

6. Conclusion

This study makes a substantial contribution to the advancement of q-rung orthopair fuzzy sets by introducing Yager t-norms and t-conorms, which serve as powerful mechanisms for managing uncertainty and imprecision in complex decision-making contexts. We have systematically defined the core operations of these aggregation tools, elucidating their underlying principles and establishing a strong theoretical foundation for their practical deployment. A central innovation of our work is the development of two novel aggregation operators—q-ROFGPOPRYWA and q-ROFGPOPRYWG—which offer enhanced flexibility and effectiveness in aggregating q-rung orthopair fuzzy numbers. These operators significantly expand the methodological arsenal available to decision scientists and practitioners working in uncertain environments. fImportantly, the applicability and robustness of the proposed methods were validated through a real-world case study focused on sustainable urban development. By integrating artificial intelligence with a Multi-Criteria Decision-Making (MCDM) framework, we demonstrated the practical value and reliability of our Yager-based aggregation operators in addressing real-life challenges. In summary, this research fills a critical gap in the fuzzy set literature while introducing innovative tools that have the potential to transform decision-making under uncertainty. The proposed Yager aggregation methods open new pathways for future exploration and application across diverse domains requiring nuanced and resilient decision-support systems.

6.1 Limitations of the Proposed Study

While this study offers meaningful insights, several limitations should be acknowledged. Firstly, the scope of the analysis may constrain its generalizability, as it emphasizes specific features of q-rung orthopair fuzzy sets and particular application domains. Certain assumptions and model simplifications may have reduced the ability to fully capture real-world complexities. The efficacy of the proposed methodologies is contingent upon the availability and quality of input data, which may be inconsistent or limited in some contexts. Furthermore, the algorithms introduced could present computational challenges, particularly in terms of processing time and resource requirements. The evaluation criteria employed, while relevant, may not encompass the full spectrum of factors necessary for comprehensive decision-making. Practical implementation might also necessitate a level of domain-specific expertise, which could hinder broader adoption. Additionally, the findings may exhibit limited external validity due to their reliance on specific datasets and contextual parameters. A lack of extensive comparison with alternative methodologies may obscure a clear understanding of the relative strengths and weaknesses of the proposed approach. Lastly, the study does not fully explore future uncertainties or ethical considerations, which may influence the applicability and societal impact of the proposed techniques.

6.2 Future Research Directions

Future research can build upon the current study in several promising directions. First, extending Yager's operations to encompass other types of fuzzy numbers and sets—such as intuitionistic, hesitant, or interval-valued fuzzy sets—would demonstrate their adaptability across diverse decision-making contexts. Enhancing the proposed aggregation operators to accommodate a greater number of criteria, improve computational efficiency, and support hybrid frameworks could significantly broaden their applicability. Moreover, adapting the multi-criteria decision-making (MCDM) framework to dynamic and real-time environments, incorporating mechanisms for advanced uncertainty handling, and integrating machine learning techniques would further improve decision accuracy and responsiveness. Expanding empirical validation through diverse case studies in sectors such as healthcare, finance, transportation, and environmental management would also enhance the external validity and global relevance of the methodology. The development of user-friendly software tools and decision-support systems can facilitate practical implementation, while fostering interdisciplinary collaboration may yield novel insights, encourage cross-sector innovations, and enhance the societal impact of the proposed approach.

Acknowledgement

Not Applicable

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] Peng, Z. R., Lu, K. F., Liu, Y., & Zhai, W. (2023). The pathway of urban planning ai: From planning support to plan-making. *Journal of Planning Education and Research*. https://doi.org/10.1177/0739456X231180568
[2] Allam, Z., & Dhunny, Z. A. (2019). On big data, artificial intelligence and smart cities. *Cities*, *89*, 80–91. https://doi.org/10.1016/j.cities.2019.01.032
[3] Petchimuthu, S., Palpandi, B., & Senapati, T. (2024). Exploring pharmacological therapies through complex q-rung picture fuzzy aczel–alsina prioritized ordered operators in adverse drug reaction analysis. *Engineering Applications of Artificial Intelligence*, *133*, 107996. https://doi.org/10.1016/j.engappai.2024.107996
[4] Petchimuthu, S., & Palpandi, B. (2025). Sustainable urban innovation and resilience: Artificial intelligence and q-rung orthopair fuzzy expologarithmic framework. *Spectrum of Decision Making and Applications*, 2(1), 242–267.

[5] Kamacı, H., & Petchimuthu, S. (2022). Soergel distance measures for q-rung orthopair fuzzy sets and their applications. In *Q-rung orthopair fuzzy sets: Theory and applications* (pp. 67–107). Springer. https://doi.org/10.1007/978-3-030-89721-1_3
[6] Kamacı, H., Palpandi, B., & Petchimuthu, S. (2025). M-polar n-soft set and its application in multi-criteria decision-making. *Computational and Applied Mathematics*, *44*, 64. https://doi.org/10.1007/s40314-024-02123-4
[7] Seikh, M. R., & Mandal, U. (2022). Q-rung orthopair fuzzy frank aggregation operators and their application in multiple attribute decision-making with unknown attribute weights. *Granular Computing*, 1–22. https://doi.org/10.1007/s41066-021-00290-2
[8] Riaz, M., et al. (2020). A robust q-rung orthopair fuzzy information aggregation using einstein operations with application to sustainable energy planning decision management. *Energies*, *13*(9), 2155. https://doi.org/10.3390/en13092155
[9] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
[10] Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *33*(1), 37–45. https://doi.org/10.1016/0165-0114(89)90226-7
[11] Seikh, M. R., & Mandal, U. (2023). Q-rung orthopair fuzzy archimedean aggregation operators: Application in the site selection for software operating units. *Symmetry*, *15*(9), 1680. https://doi.org/10.3390/sym15091680
[12] Senapati, T., Simic, V., Saha, A., Dobrodolac, M., Rong, Y., & Tirkolaee, E. B. (2023). Intuitionistic fuzzy power aczel-alsina model for prioritization of sustainable transportation sharing practices. *Engineering Applications of Artificial Intelligence*, *119*, 105716. https://doi.org/10.1016/j.engappai.2022.105716
[13] Gohain, B., Chutia, R., & Dutta, P. (2022). Distance measure on intuitionistic fuzzy sets and its application in decision-making, pattern recognition, and clustering problems. *International Journal of Intelligent Systems*, *37*(3), 2458–2501. https://doi.org/10.1002/int.22716
[14] Ke, Y., Tang, H., Liu, M., & Qi, X. (2022). A hybrid decision-making framework for photovoltaic poverty alleviation project site selection under intuitionistic fuzzy environment. *Energy Reports*, *8*, 8844–8856. https://doi.org/10.1016/j.egyr.2022.06.007
[15] Wan, S., & Yi, Z. (2016). Power average of trapezoidal intuitionistic fuzzy numbers using strict t-norms and t-conorms. *IEEE Transactions on Fuzzy Systems*, *24*(5), 1035–1047. https://doi.org/10.1109/TFUZZ.2015.2492543
[16] Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965. https://doi.org/10.1109/TFUZZ.2013.2241045
[17] Akram, M., Dudek, W. A., & Dar, J. M. (2019). Pythagorean dombi fuzzy aggregation operators with application in multicriteria decision-making. *International Journal of Intelligent Systems*, *34*(11), 3000–3019. https://doi.org/10.1002/int.22180
[18] Shahzadi, G., Akram, M., & Al-Kenani, A. N. (2020). Decision-making approach under pythagorean fuzzy yager weighted operators. *Mathematics*, *8*(1), 70. https://doi.org/10.3390/math8010070
[19] Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *25*(5), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005
[20] Wang, J., Wei, G., Wei, C., & Wei, Y. (2020). Mabac method for multiple attribute group decision making under q-rung orthopair fuzzy environment. *Defence Technology*, *16*(1), 208–216. https://doi.org/10.1016/j.dt.2019.05.005
[21] Wang, J., Zhang, R., Zhu, X., Zhou, Z., Shang, X., & Li, W. (2019). Some q-rung orthopair fuzzy muirhead means with their application to multi-attribute group decision making. *Journal of Intelligent and Fuzzy Systems*, *36*(2), 1599–1614. https://doi.org/10.3233/JIFS-181513

[22] Wang, J., Wei, G., Lu, J., Alsaadi, F. E., Hayat, T., Wei, C., & Zhang, Y. (2019). Some q-rung orthopair fuzzy hamy mean operators in multiple attribute decision-making and their application to enterprise resource planning systems selection. *International Journal of Intelligent Systems*, *34*(10), 2429–2458. https://doi.org/10.1002/int.22183

[23] Kausar, R., Farid, H. M. A., Riaz, M., & Bilgin, N. G. (2023). Innovative codas algorithm for q-rung orthopair fuzzy information and cancer risk assessment. *Symmetry*, *15*(1), 205. https://doi.org/10.3390/sym15010205

[24] Liu, P., & Liu, J. (2018). Some q-rung orthopair fuzzy bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems*, *33*(2), 315–347. https://doi.org/10.1002/int.21940

[25] Xing, Y., Zhang, R., Wang, J., Bai, K., & Xue, J. (2020). A new multi-criteria group decision-making approach based on q-rung orthopair fuzzy interaction hamy mean operators. *Neural Computing and Applications*, *32*, 7465–7488. https://doi.org/10.1007/s00521-019-04054-3

[26] Yager, R. R. (2001). The power average operator. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, *31*(6), 724–731. https://doi.org/10.1109/3468.983420

[27] Yager, R. R. (2009). Prioritized owa aggregation. *Fuzzy Optimization and Decision Making*, *8*, 245–262. https://doi.org/10.1007/s10700-008-9047-0

[28] Vargas-Hernández, J. G., Pallagst, K., & Zdunek-Wielgo laska, J. (2023). Urban green spaces as a component of an ecosystem. In *Sustainable development and environmental stewardship: Global initiatives towards engaged sustainability* (pp. 165–198). Springer International Publishing. https://doi.org/10.1007/978-3-030-87965-5_8

[29] Khan, M. R., Wang, H., Ullah, K., & Karamti, H. (2022). Construction material selection by using multi-attribute decision making based on q-rung orthopair fuzzy aczel–alsina aggregation operators. *Applied Sciences*, *12*(17), 8537. https://doi.org/10.3390/app12178537

[30] Green, D., O'Donnell, E., Johnson, M., Slater, L., Thorne, C., Zheng, S., & Boothroyd, R. J. (2021). Green infrastructure: The future of urban flood risk management? *Wiley Interdisciplinary Reviews: Water*, *8*(6), e1560. https://doi.org/10.1002/wat2.1560

[31] Johnson, C., Matthews, T., Burke, M., & Jones, D. (2022). Acknowledgement of environmental concerns in transport infrastructure planning: A systematic review of the literature.

[32] Shokoohyar, S. S., Jafari Gorizi, A., Ghomi, V., Liang, W., & Kim, H. J. (2022). Sustainable transportation in practice: A systematic quantitative review of case studies. *Sustainability*, *14*(5), 2617. https://doi.org/10.3390/su14052617

[33] Zhu, J., Xie, N., Cai, Z., Tang, W., & Chen, X. (2023). A comprehensive review of shared mobility for sustainable transportation systems. *International Journal of Sustainable Transportation*, *17*(5), 527–551. https://doi.org/10.1080/15568318.2022.2059872

[34] Cuthill, N., Cao, M., Liu, Y., Gao, X., & Zhang, Y. (2019). The association between urban public transport infrastructure and social equity and spatial accessibility within the urban environment: An investigation of tramlink in london. *Sustainability*, *11*(5), 1229. https://doi.org/10.3390/su11051229

[35] Dubbeling, M., & De Zeeuw, H. (2011). Urban agriculture and climate change adaptation: Ensuring food security through adaptation. In *Resilient cities: Cities and adaptation to climate change-proceedings of the global forum 2010* (pp. 441–449). Springer Netherlands. https://doi.org/10.1007/978-94-007-0785-6_44

[36] Korir, S. C., Rotich, J. K., & Mining, P. (2015). Urban agriculture and food security in developing countries: A case study of eldoret municipality, kenya. *International Journal of Agricultural Policy and Research*, *3*(4), 135–141. https://doi.org/10.15739/IJAPR.2015.013

[37] Warren, E., Hawkesworth, S., & Knai, C. (2015). Investigating the association between urban agriculture and food security, dietary diversity, and nutritional status: A systematic literature review. *Food Policy*, *53*, 54–66. https://doi.org/10.1016/j.foodpol.2015.03.001

[38] Tapia, C., Randall, L., Wang, S., & Borges, L. A. (2021). Monitoring the contribution of urban agriculture to urban sustainability: An indicator-based framework. *Sustainable Cities and Society*, *74*, 103130. https://doi.org/10.1016/j.scs.2021.103130

[39] Adeyemo, R., & Kuhlmann, F. (2009). Resource use efficiency in urban agriculture in southwestern nigeria. *Tropicultura*, *27*(1), 49–53. https://www.tropicultura.org/text/v27n1/49.pdf

[40] Rong, Y., Li, Q., & Pei, Z. (2020). A novel q-rung orthopair fuzzy multi-attribute group decision-making approach based on schweizer-sklar operations and improved copras method. *Proceedings of the 4th International Conference on Computer Science and Application Engineering*, 1–6.